Collection, analysis, and utilisation of detailed recorded data on students' mathematical difficulties in a university mathematics support centre.

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Publications

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Abbreviations

A-level	Advanced level
CAO	Central Applications Office
CER	Chief Examiner's Report
GCE	General Certificate of Education
GCSE	General Certificate of Secondary Education
HEA	Higher Education Authority
HEI	Higher Education Institution
LCE	Leaving Certificate Examination
MSC	Mathematics Support Centre (Maths Support Centre)
SEC	State Examinations Commission
UCAS	Universities and Colleges Admissions Service
UCD	University College Dublin
UK	United Kingdom

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Chapter 1 Introduction

The report, 'Tackling the mathematics problem' (Howson et al., 1995) made it clear that serious concerns existed amongst those in higher education in the UK in relation to the mathematical preparedness of undergraduate students. The report highlighted three main areas of difficulty: a lack of fluency in numerical and algebraic calculations, a decline in analytical ability, and a decrease in the understanding of the importance of precision and proof in mathematics. It recommended that within a climate of widening access to university, those involved in higher education should re-consider their provision for students entering third-level education.

The influential report 'Measuring the mathematics problem' (Hawkes & Savage, 2000) identified major issues facing mathematics and engineering departments in higher education institutions in the United Kingdom (UK). These related to the level of students' preparedness for mathematics based degree courses. The authors noted that a critical reduction in students' ability with basic mathematical skills had been indicated by diagnostic testing. The report made a number of recommendations, one of which was that '*prompt and effective support should be available to students whose mathematical background is found wanting*' (2000, p.iv). The provision of mathematics support was viewed by many as a means of addressing this issue.

Although mathematics support at third-level may have existed earlier it was only in the 1990s that mathematics support centres (MSC) were introduced on a larger scale

(Beveridge & Bhanot, 1994). Today they play a pivotal role in providing mathematics and statistics support in higher education (Perkin, Croft, & Lawson, 2013; Cronin, Cole, Clancy, Breen, & O'Sé, 2016). Recognition of the contribution of those working in mathematics support was seen in the award of Gold Medals in 2016 to Professor Tony Croft and Professor Duncan Lawson by the Institute of Mathematics and Applications (IMA) for their outstanding contribution to the improvement of the teaching of mathematics.

University College Dublin (UCD) is the largest university in Ireland with approximately 25,000 students on campus. Opened in 2004, the UCD Maths Support Centre (MSC) is a mathematics and statistics drop-in centre where support is provided, primarily in the form of one-to-one or small group tuition, free of charge, to UCD students. This initially applied to students at all levels in the university but since semester 1 2016 only students in modules at levels 0, 1 or 2, have access. The MSC is now embedded as a university-wide resource and is funded centrally by the university. It has grown steadily over the last fifteen years and in 2016/2017, saw 5,252 visits from 1,380 distinct students. It opens for 39 hours per week and is currently staffed by a full-time manager and part-time postgraduate tutors.

Experience in managing the MSC in UCD, extending over six years from September 2007 to August 2013, sparked my interest in the *mathematical difficulties* experienced by students in third-level education. In January 2009, whilst manager of the MSC, I designed and oversaw the development of a web application to maintain an electronic record of each UCD student visit to the MSC. For each visit, the date and length of the visit, the module for which the student required help, their programme of study, and other background information, were recorded on the database. On completion of each student visit to the MSC, the assisting tutor inputted, details of the *mathematical difficulties* experienced by the student, to the database. Once entered on the database, the module lecturer was then able to access these anonymous tutor entries electronically if he or she wished, and is presently, also sent a weekly email with the information. Whilst manager of the MSC, I approached a number of lecturers to evaluate the usefulness or otherwise of this feedback. However, student attendance at the MSC increased significantly over this time and management and tutoring duties left little time to pursue this research area in further detail. Therefore, having retired as manager of the MSC, I was keen to undertake this research study.

The current manager updated the application, in 2015, and received permission to link it directly to the UCD central registry database (Cronin & Meehan, 2015).

This study focused on identifying and recording areas of *mathematical difficulty* encountered by students in the *lived experience* of a mathematics support centre. Data has been collected on the MSC database for four years prior to the academic year 2013-2014 and the original intention was to analyse these data in order to identify areas of *mathematical difficulty*. The lack of detail in the data, however, meant this was not possible. In order to obtain in-depth information on the *mathematical difficulties* experienced by students attending the centre a more comprehensive data collection process was undertaken. This involved eight weeks of intensive collaborative work with the MSC tutors including the coding of all *mathematical difficulties*. A further aim of the research was to ascertain the benefits or otherwise of the feedback generated on MSC students' visits and accessible, in real time, to all lecturers within the School of Mathematics and Statistics. To this end, questions were posed in interviews with 13 lecturers. These interviews were conducted on three occasions, at the beginning, during and just after the eight weeks of the research project. A focus group was organised with the MSC tutors at the end of the academic year 2014-2015 to ascertain their views on possible improvements to the data entry process.

Using these data, the following research questions were addressed:

- *What are the common mathematical difficulties which students present with at the Maths Support Centre from (a) across modules, and (b) within a given module?*

2. *What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?*

3. *In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?*

4. *What feedback, if any, would be most beneficial for lecturers to receive on their students' visits to an MSC?*

The first question investigated the *mathematical difficulties* exhibited by students who attended the UCD Mathematics Support Centre (MSC) over an eight-week period in Semester 1, 2014/2015. These *mathematical difficulties* were recorded online by the tutors for each individual student visit. The students' difficulties were coded under thirty-one mathematical topics which were further organised under groups as follows: *Algebra, Calculus, Applied Mathematics, Statistics, Advanced Mathematics* and *Miscellaneous.* Each group was analysed to show the major areas of *mathematical difficulty* exhibited by the students. These were examined in further detail to show the level of the modules from which students exhibited the difficulties and where similar difficulties were evident in a number of different modules.

An ongoing cause for concern in higher education was the poor mathematical skills of incoming students. For this reason, mathematical diagnostic tests are issued to the whole cohort of first year students in specific programmes by many third-level institutions. These tests are beneficial in assessing students' prior mathematical knowledge and skills, identifying those with high probability of failure and selecting students for further help. This study used a different approach by examining the *mathematical difficulties* exhibited by students in the *lived experience* of students attending a mathematics support centre over an eight-week period. It provides information not only on the nature of students' weaknesses in relation to prior knowledge but also where difficulties arose with the module content.

The second research question exploring the use of available resources examined to what extent these *mathematical* *difficulties* related to *Module Content* rather than *Prior Knowledge*. Discussion of these findings asks if they challenge the existence of mathematics support centres (MSC), since the original motivation for these centres was to overcome problems in relation to the mathematical under-preparedness of students on entry to third-level education.

The data collected contained information on the student number, the frequency of visits, the module the student sought assistance for and a detailed outline of the *mathematical difficulties* exhibited by the student. These difficulties were given an initial coding by the tutors for each student visit and the coded details were uploaded online to a database. The third research question asks if the knowledge gained from the data including the tutor input to the process would be beneficial in providing information to allow the more efficient functioning of the MSC for example, in the use of group classes such as those run for *Hot Topics*. Overcrowding has been an issue in the centre, was there evidence from the data to show a disproportionate drain on the centre's resources and suggest possible strategies to increase the efficacy of the MSC? Also discussed here is the focus group with tutors examining methods to make the process of uploading the *mathematical difficulties* exhibited by a student more efficient yet maintaining useful information for the lecturer.

Mathematics Support Centres were originally run by lecturers and they were therefore aware of the difficulties with the mathematical concepts and skills for which their students sought help. Centres more normally these days, employ PhD students or other teachers to tutor in mathematics support centres and therefore, lecturers may no longer have access to the same data as originally available to them. In an attempt to bridge this gap in the supply of information, the UCD MSC made the tutor entries on the *mathematical difficulties* exhibited by students attending the MSC, available online to the module lecturer, if they wish to receive them. Information gathered from interviews with thirteen lecturers, each interviewed on three separate occasions in Semester 1, 2014/2015, was analysed to ascertain what feedback from the MSC was beneficial for lecturers to receive and in what form it should be displayed.

This study is presented in six chapters.

In Chapter 2, relevant literature relating to the following areas is discussed: similarities and contrasts in mathematics education at second-level in the UK and Ireland, mathematical concerns in the transition to third-level, and, the introduction and evaluation of MSCs.

The four stages of the data collection process are presented in Chapter 3. The first stage explains the original data collection. Following this, the initial coding of the data and the training of the tutors are outlined and the research questions are stated. The second stage discusses how the coding, developed in stage 1, was refined. The third stage is a description of the pilot study. The fourth stage consists of the main study, conducted over a period of eight weeks, and is described with an analysis of the data presented. Analysis of the focus group held with tutors is summarised and finally analysis of responses from interviews with lecturers follows.

In Chapter 4 detailed findings of the data collected in stage 4 as described above is presented and results from the tutor focus group are provided and this is followed by the results of the interviews with lecturers.

Chapter 5 discusses the findings in relation to previous research, and highlights the key issues concerning data collection and analysis for the MSC and benefits of feedback for MSC and lecturers.

Chapter 6 provides a summary of the findings, and ends with a number of recommendations.

Chapter 2 Literature Review

For the past thirty years, many lecturers, teachers, educational policy makers and researchers worldwide have struggled to find explanations and solutions for what has become known as the *mathematics problem*, which is that a number of entrants to third-level education appear to be inadequately prepared mathematically for some requirements of third-level curricula. The increasingly quantitative nature of many third-level programmes requires a knowledge of mathematics and statistics which was previously the domain of mathematicians, physicists and engineers. Programmes such as medicine, psychology, social science and geography, for example, now demand a level of mathematics not required in former years.

Many theories have been proposed to explain the mathematics problem. Although, previous literature had covered a wide variety of such theories, this review focused on five major themes. These themes were: similarities and contrasts in the Irish and United Kingdom (UK) systems of education; earlier literature describing the mathematics problem; lecturers' and students' perceptions of the students' difficulties; possible factors influencing the problem in the Irish context; and, a number of responses to the mathematics problem.

2.1 Similarities and contrasts in second-level education in the UK and Ireland

2.1.1 Second-level education in the UK

Second-level education in the UK is taken by students from eleven to eighteen years of age and is compulsory up to sixteen years of age. The Scottish post-16 system emphasises breadth across a range of subjects, normally five, while the English, Welsh and Northern Irish systems require greater depth of education across a smaller range of subjects, normally one to four A-levels. Students in the UK (other than Scotland) typically take the General Certificate of Secondary Education (GCSE) at sixteen years of age and a number of students continue their studies to take A-level examinations (Advanced level) at eighteen. There are a number of separate awarding bodies for A-level examinations and for this reason the mathematical content between regions may have varied. Each examination covered the same four core areas but options were then chosen for a further two. However, significant changes have been made in A-level mathematics curriculum and assessment in England beginning in 2017. According to the Smith Report (2017) the new A-levels will have the same content, students study pure mathematics as two-thirds of the curriculum, one third will be devoted to statistics and mechanics and the examination system changed from modular to linear assessment. Smith stated that these changes reflected approaches from universities for a common compulsory curriculum. Also the new curriculum specification has removed a great deal of variation between qualifications. However, Wales and

Northern Ireland have not yet adopted these changes and continue with the old system of AS and A levels.

Approximately 15% of students (Hodgen, Pepper, Sturman & Ruddock, 2010) in England continue their mathematics studies after GCSE although the UK government has recently considered making the study of Mathematics and English compulsory up to eighteen years (Hudson, 2006). The Smith Report (2017) was a review of the mathematical education in England experienced by 16-18 years old students, including the issue of most or all of them continuing mathematics education to 18. The report did not recommend the immediate compulsory study of mathematics beyond 16 as they found both the range of pathways was limited as was the capacity to deliver the necessary teaching required at the time of the report. However, it was suggested it might be possible to achieve this at some later date, suggesting ten years as a possible implementation date.

Entry to university in England has various routes some through A-level, some through BTEC qualifications. Gicheva and Petrie (2018) showed the percentage of those applying for university in 2016 with A-levels alone as 54% and through BTEC qualifications as 26% including 18% applying holding only BTEC and 8% studying a mixture of A-levels and BTEC qualifications. Twenty percent applied with a different qualification.

2.1.2 Second-level education in Ireland

Second-level education in Ireland, often referred to as post-primary education, is normally taken by students from the ages of twelve to nineteen years. It consists of a three-year “junior cycle” and a two-year “senior cycle”, frequently with a one-year “transition year” in between. At the end of the final two years of Senior Cycle students take a State Examination known as the Leaving Certificate (LC) in six to eight subjects.

There is a single awarding body for this examination known as the State Examination Commission (SEC). Subjects are offered at Higher Level, Ordinary Level and, in certain cases, Foundation Level. Currently, grades in the Leaving Certificate are defined (Central Applications Office, 2015), by the scales as shown in Table 2.1 below.

Applicants for places on programmes in higher education in Ireland must satisfy the minimum requirements for their course. Entry to HEIs (Higher Education Institutions) is competitive and is based on the results in the Leaving Certificate Examination (LCE). Consequently, these are high-stakes examinations as they act as a gatekeeper for third-level education in Ireland.

Table 2.1 Grading scale in Leaving Certificate Examination after 2016

Percentage Result	**Grade**
90 - 100%	H1/O1
80 - 89%	H2/O2
70 - 79%	H3/O3
60 - 69%	H4/O4
50 - 59%	H5/O5
40 – 49%	H6/O6
30 – 39%	H7/O7
0 – 29%	H8/O8

The system for entry to third-level, which is commonly referred to as the *points system*, is administered by the Central Applications Office (CAO). All universities, institutes of technology, colleges of education and many private and partially publicly funded HEIs, use the CAO to select applicants. Students specify their preference for higher education courses to the CAO and places are allocated on the basis of a rank order of students on a points scale (Hyland, 2011).

The points a student earns are based solely on their best results in six subjects with the maximum number of points being 600. In 2012, a scheme was introduced to encourage more students to take the Higher Level paper in mathematics. A bonus of 25 points is awarded to a student passing the Higher Level paper irrespective of the grade achieved. Thus for students, since 2012, the maximum number of points which can be awarded is 625 (Central Applications Office, 2012). Approximately 75% of places at higher education institutions in Ireland are based on CAO points achieved. In other words, 75% of places are based solely on a student's academic performance in a single examination at the end of their post-primary education. Although mathematics is not a compulsory subject for the LCE, almost all students in Ireland study it for the duration of their post-primary education. Mathematics is offered at three levels for the LCE - Foundation, Ordinary and Higher Level. The vast majority of students will enter third-level institutions in Ireland with, at the very minimum, a pass grade at the Ordinary Level, although some undergraduate programmes, for example in mathematics or engineering, may require the student to have received a B- or C-grade or higher in the

Higher Level examination, or an equivalent international qualification. Students are aware that Higher Level mathematics covers a wide syllabus and is therefore very time consuming.

In previous years, students may have decided against taking the Higher Mathematics and transferred to the Ordinary Level, to allow them to devote extra time to other subjects, where they believed they would more easily obtain the necessary points for their desired course at university. Even when a student chose to study the Higher Level mathematics the fact that there was choice of questions on each paper suggests that students could avoid studying and indeed some teachers may have avoided teaching, quite large areas of the syllabus (Lubienski, 2011).

A new approach and syllabus for mathematics at post-primary level, previously named *Project Maths* was introduced in Ireland in 2010. In terms of content, among the changes in syllabus at LCE were the following:

- *an increase in the proportion of the syllabus dealing with statistics and probability;*
- *the removal of the study of vectors and matrices;*
- *changes to the material on functions and calculus; and*
- *the choice of questions has been removed from the papers.*

The number of students taking the Higher Level examination has increased. Among the LCE mathematics candidates for 2017, a record 36% have registered to sit the Higher paper, up from about 20% in 2011.

Applied mathematics is a separate mathematics subject for the LCE with a syllabus based on mathematical physics. Not

all schools offer this subject for the LCE and a number of students study it externally as an e
xtra subject. It is assessed at Ordinary and Higher Levels of the LCE. In June 2015, approximately 4% of those students taking mathematics for the LCE also sat the applied mathematics paper, this increased to approximately 12% if we considered only those taking Higher Level mathematics. Student numbers taking the LCE in applied mathematics have increased slightly with uptake just over 5% last year.

2.1.3 Summary of contrasts and similarities- UK and Ireland

An important research report by Hodgen, Peppers, Sturman, and Ruddock, (2010), funded by the Nuffield Foundation, highlighted similarities and differences in mathematics education in upper-secondary education in 24 countries. According to this report, unlike most countries reviewed, mathematics was not a compulsory subject in upper-secondary general education in the UK or Ireland. However, it indicated there was a considerable variation in the percentage of students studying mathematics at this level in both countries. In Ireland, they stated that students took a range of subjects for the final examination almost universally choosing mathematics and this reflected the high level of competition for university places where mathematics at this level is a requirement for many third-level studies. Table 6 (2010, p.38) showed that, other than for Scotland which has a slightly higher rate, the percentages of those studying mathematics at this level in the UK is less than 20%.

Both Ireland and England have made recent changes to the curriculum and assessment processes for the final secondary level examination. These changes have not taken place in Wales or Northern Ireland. It is too soon to tell if these changes will affect the numbers taking these examinations in England as the first examination took place in 2018. The changes in Ireland have indicated that the numbers taking advanced mathematics in Ireland have increased from low (0-15%) as seen in (Hodgen, Peppers, Sturman, & Ruddock, 2010, p.38) to over 31% as seen in the State Examination Statistics (State Examination Commission, 2018).

A further distinction is the number of examining boards for the examination, one in Ireland and a number of different boards in the UK. However, in England differences in these examinations have been largely reduced due to the new approach to A-level examinations.

In England, applied mathematics (mechanics) is included in the new linear syllabus for A-level whereas in Ireland it is a separate subject with low uptake in numbers. It is optional, at the present time, in A-levels for Wales and Northern Ireland.

2.2 Background to the mathematics problem

Problems of transition from second-level to third-level education were not confined to Ireland alone. De Guzman, Hodgson, Robert, and Villani (1998) described widespread difficulties throughout a number of European countries. Even countries, such as Korea and Taiwan, often regarded internationally as top performers in mathematical attainment, saw themselves in mathematical crisis (Hoyles, Morgan, & Woodhouse, 1999; The National Academies of Sciences, 2015). Further reports in the UK added to this growing concern (Hawkes & Savage, 1999; Hodgen, McAlinden, & Tomei, 2014).

2.2.1 Situation in the United Kingdom

Issues such as the increasingly heterogeneous nature of student qualifications, variable entrance routes to higher education, radical modification of second-level mathematics curricula and lowering of admission standards to allow for expansion in admission numbers, were all highlighted as potential factors influencing the students' mathematical knowledge at entry to third-level education in the UK, (Royal Society/Joint Mathematical Council, 1997; Hawkes & Savage, 2000: Hodgen, McAlinden, & Tomei, 2014; Smith, 2017).

Two early reports, in relation to the *mathematics problem* in the UK, highlighted the effects of changes in mathematics education at secondary level. The first report, that of the Dainton Committee (1968), considered the consequential effect on the country's economic development resulting from the diminishing numbers taking science subjects at 'A-level'. The need for a skilled workforce amongst other reasons resulted in compulsory education in the UK being raised to 16 years of age in September 1972. The second, by Cockcroft (1982), expressed disquiet on the lack of basic computational skills, the multiplicity of mathematics syllabi and the lack of communication between schools and higher education. The scarcity of qualified mathematics teachers was a difficulty highlighted in both of these papers.

A paper entitled, 'Tackling the mathematics problem' (Howson et al., 1995) appears to be the earliest reference to the '*mathematics problem*'. A working group, commissioned by the London Mathematical Society, issued this report on behalf of three eminent organisations: The London Mathematical Society, the Institute of Mathematics and its Applications, and the Royal Statistical Society. The authors acknowledged the importance of numeracy and accepted the key role that mathematics played in a modern economy. They cited additional reports, published in the same year by the Engineering Council (Sutherland & Pozzi, 1995) and the Institute of Mathematics and its Applications (1995), which had also expressed serious concerns in relation to the decline in the mathematical preparedness of incoming undergraduates. Howson et al., (1995), found that this problem related not only to students who completed their mathematical studies at sixteen but existed even in universities which selected the highest qualified entrants. They suggested that these reports highlighted a reduction, not only in analytical ability, but also in the acceptance by these students, of the essential nature of precision and proof in mathematics with many new undergraduates lacking fluency and accuracy in algebraic calculations. They proposed, as possible causes, the introduction at second-level of time-consuming activities such as investigations, problem-solving and data surveys with a loss of core proficiency, a reduction in the time allocated to mathematics classes in school and a change in emphasis away from technical skills. The authors stated that:

> *'Progress in mastering mathematics depends on reducing familiar laborious processes to automatic mental routines, which no longer require conscious thought; this then creates space to allow the learner to concentrate on new and unfamiliar ideas' (1995, p.11).*

The Howson report (1995) also noted that much of the evidence from higher education was inferred and therefore difficult to substantiate. However, they suggested, it reflected the judgement of almost all those they had consulted. As evidence, they referred to a study by Coe and Ruthven (1994) concerned with proof which demonstrated that only two students, out of a cohort of sixty, understood what was required to prove a given hypothesis. Howson et al., (1995), argued that a decline at school level was further reinforced by results from the national Olympiads over the previous 6 years. In conclusion, the report suggested the need for an increased emphasis on important basic mathematics and relevant mathematical skills. Among their recommendations they emphasized the importance of higher education involvement at all levels of mathematics' educational decisions from primary to university level.

Comments from studies in the mid nineteen-nineties considered long-term findings of *mathematical difficulties* existing for students in third-level education. The report by Sutherland and Pozzi (1995), mentioned above, noted that a majority of the engineering lecturers they had surveyed stated that the mathematical knowledge of first year undergraduate engineering students had weakened over the previous ten years and more than half of these lecturers had surmised that this was undermining the quality of their degrees. Lawson (1997), in a long-term study comparing the results of the same diagnostic test obtained by students with "A level" qualifications over seven years (1991-1997), demonstrated that there was little difference between those with A-level grade C in 1997 when compared with those with

A-level grade N in the 1991 examination. Grade N was a grade awarded to those who just failed to achieve a pass in the examination. His report revealed that an unexpectedly large portion of those, even with A-level grade C, had difficulty with routine tasks covering linear functions and indices. A separate report of the Royal Society/Joint Mathematical Council, (1997), highlighted algebra as a major cause of concern and called for a review of the teaching and learning approach to algebra in pre-university mathematics, Lawson (1997) supported the provision of this review as a matter of urgency.

The report 'Measuring the Mathematics Problem' (Hawkes & Savage, 2000) was aimed at those who teach students at third-level and those involved with admissions and with responsibility for setting A-level examinations. It was published under the auspices of The Learning and Teaching Support Network, the Institute of Mathematics and its Applications, the London Mathematical Society, and the Engineering Council. This report reflected on the history of changes in A-level mathematics over the period from 1960-2000 and their effect on higher education. They stated that the 1960's A-level mathematics was controlled by the universities and served their needs. They identified this time as the *golden age* for A-level mathematics, when students acquired the sound mathematical knowledge and understanding deemed necessary for higher education. The first major change, they explained, came about with the introduction of statistics in the 1970s. Three mathematics courses were then available to students:

- *Pure and Mechanics,*

- *Pure and Statistics, and*
- *Pure and Applied.*

The main difficulty then arising at third-level, they suggested, was for students who had taken only the pure and statistics A-level, but needed to study mechanics. However, since the standard of their pure mathematics was high, it was possible for these students to cope. The authors explained that the major difficulties arose when higher education in the mid-eighties, lost responsibility for A-level examinations and when GCE examinations were replaced by GCSE where the concept of proof, technical skills and understanding of algebra declined.

A further report relating to A-level examinations by Kitchen (1999) using data collected from a number of sources, stated that:

> *'while the standards of the mathematics 'A-level' as a whole may not have declined, the standard of attainment of that content deemed most important for progression to higher education is almost certainly less, especially at lower grades' (1999, p.72).*

The author (1999) concluded that the diverse A-level mathematics syllabi resulted in a reduction in the pure mathematics content for many A-level examinations and particularly, a decrease in the demand for algebraic fluency. Other causes suggested, were the lowering of entry standards in an effort to increase numbers at third-level and the reliance by lecturers on students' mastery of algebra, calculus and trigonometry.

By 2000 there appears to be little progress in improving the standard of students entering higher education. The serious decline in mathematics skills and problems with levels of preparedness for mathematics programmes in higher education had been accepted and reports were more concerned at finding solutions. The report, 'Measuring the Mathematics Problem' (Hawkes & Savage, 2000), as described earlier, not only provided evidence of students' lack of basic mathematical skills but also described effective measures implemented in higher education at that time. They proposed a number of recommendations, including the use of diagnostic tests. This they recommended as a two-stage process, testing and follow-up. Further proposals were the provision of effective supports and the establishment of a standing committee for mathematics to include members from all relevant educational sectors.

Four years later Smith (2004) issued a report, mainly in reference to the situation in England, in which he stated that he found it deeply disturbing that so many stakeholders involved in mathematics education in England considered there was a crisis in the teaching and learning of mathematics. Smith expressed concern in three main areas:

- *Failure of the curriculum and qualifications framework on two counts, that it failed to meet the requirements of higher education and employers and failed to motivate students to continue the study of mathematics after sixteen.*
- *Shortage of specialist mathematics teachers with subsequent adverse effect on students' learning, and*
- *The lack of support infrastructure to provide continuing professional development and resources for those involved in the teaching of mathematics.*

The author made a number of recommendations aimed at averting the perceived crisis in mathematics education. Among these was the recommendation (2004, 4.4, p.154) that increased time be allocated to the mathematics curriculum to allow for the reinforcement of core skills, such as fluency in algebra, reasoning about geometrical properties and a realisation of the key importance of statistics in its own right with the desirability of its integration with other subjects.

Six years later a report, 'Responding to the Mathematics Problem' (Marr, & Grove, 2010), contained collected essays from a meeting held, at the University of St Andrews, with a number of people involved in higher education. The report considered the support measures which were implemented in various higher level institutions in response to the *mathematics problem* and the future development of these resources. Vorderman, Porkess, Budd, Dunne, and Rahman-Hart, (2011) suggested radical changes to the GCSE and the introduction of compulsory mathematics education for all students up to the age of eighteen. In the introduction to this report, Michael Gove, British Secretary of State for Education from 2010 to 2014, returning to the theme of the economy, stated in relation to the report:

> *'We must reform. We need to reform teacher training to get even more talented people into the classroom, we need a more rigorous curriculum which matches the world's best, and we need exams which equip our students for the society and economy of the future' (2011, p.iii).*

2.2.2 Situation in Ireland

A major report, issued by the Organization for Economic Co-operation and Development (OECD), and known as *Investment in Education* (Department of Education, 1965) was commissioned by the Irish government in 1962 around the same time as the Dainton report (1968) in the UK. Both were necessitated by the developing economies and the resulting requirement for a better educated workforce. The OECD report resulted in a series of Irish government measures including free education to the age of eighteen. This expanded secondary-school enrolment and graduation rates and significantly increased the demand for third-level places.

Issues concerning the lack of mathematical skills of new entrants to Higher Education Institutes (HEIs) in Ireland became evident in the 1980s, about the time of the Cockcroft report (1982) in the UK. Papers such as those by Hurley and Stynes (1985), Hurley and Stynes (1986), and O' Murchu and O' Sullivan (1982) mentioned fundamental deficiencies in the mathematics skills as evidenced by tests given to their in-coming students. The RTC study (Hurley & Stynes, 1985) related to students in a Regional Technical College (RTC). Technical colleges in Ireland offered, besides degree programmes, courses leading to a technical certificate or diploma, normally of two and three years' duration and with a lower level of entry required. The authors of the RTC report stated that the extent of the problem was particularly alarming for the non-degree students as 80% of them had failed the test. The other two reports (Hurley & Stynes, 1986; O' Murchu & O'Sullivan, 1982) addressed *mathematical* *difficulties* of students, taking science and physics degree courses. The common mathematical areas of difficulty found were surds, exponents, conversion of units and logarithms.

O'Murchu and O'Sullivan (1982) stated that they were convinced that a major element of the student difficulty arose from their lack of fundamental mathematical skills. They concluded that *'the problem was not an absence of knowledge but rather a total lack of facility with even the simplest operations.'* (1982, p.51). The cause, according to the authors, was a lack of reinforcement of elementary concepts and increasing reliance on rote memory. Hurley and Stynes (1985) remarked that in discussion with colleagues teaching mathematics at other third-level colleges, they believed that the problem existed throughout the country and that the same tests would have yielded equally poor results elsewhere.

Diagnostic testing was first introduced in the University of Limerick (UL), in 1997, possibly influenced by the reports of similar testing in Loughborough. This was part of a pilot study to explore the under-preparedness of students in Level 1 service mathematics courses and to develop new methods of dealing with the problem (O'Donoghue, 1998). Gill and O'Donoghue (2007b) using a longitudinal study of diagnostic test results, similar to that of Lawson (1997), suggested that the Leaving Certificate Ordinary Level mathematics was not an adequate preparation for service mathematics courses in UL and furthermore that grade dilution in Leaving Certificate was evident by looking at mean scores in the tests over the previous six years. In reference to two earlier studies (Lyons, Lynch, Close, Sheeran, & Boland, 2003; Murphy, 2002), the authors explained possible reasons for these difficulties and concluded that teaching at second-level in Ireland involved teaching to the examination rather than to the aims of the curriculum with an undue emphasis on the Leaving Certificate examination. Other studies (Gill & O'Donoghue, 2006b; Faulkner, Hannigan & Gill, 2010; Treacy & Faulkner, 2015) also examined long-term diagnostic testing to show further downward trends in the mathematical competency of students entering third-level.

A small number of papers served as evidence of actual *mathematical difficulties* seen at the transition from post-primary to third-level education. In 2009 a five-year review of diagnostic testing in a university in Dublin (Ní Fhloinn, 2009a), described areas where questions were invariably poorly answered. These were, in order of least well answered, fractional indices; algebraic indices; solving partially-factored cubic equations; and inequalities. Sheridan (2013) looked at failure rates on diagnostic testing and the effect of follow-on support and referenced algebra and arithmetic as two main areas of difficulty.

Difficulties were not just limited to transition from post-primary to third-level. The administration of an advanced diagnostic test, to students in an honours engineering degree programme and an approximately equal number of students in an ordinary engineering degree programme, was described by Carr, Murphy, Bowe and Ní Fhloinn (2013). The results showed 88% of students answered questions on basic differentiation correctly, but this decreased to 31% for product rule, 21% for quotient rule and to 37% for chain rule. In basic integration only 68% achieved the correct answer. Differential equations (ODEs) both first and second order caused further difficulty -1st order with 37% correct and 2nd order ODEs with only 1% correct answers provided. A low level of knowledge in matrices, with 5% of questions on matrix multiplication correctly answered, and with multiplication of complex numbers at 3% of questions correct, were other difficulties highlighted.

The Chief Examiner's Reports (CER) in Ireland provided a significant review of the performance of candidates in post-primary examinations. The reports are published in a selected number of subjects and programmes each year. Four separate reviews with a detailed analysis of the standards of students' answers to questions on the LCE paper in mathematics were produced by the chief examiner from the year 2000 to the present day.

The 2000 CER in mathematics (Eolaiolchta, 2000) stated that in the Higher Level examination, candidates' strengths were seen in procedural type questions where a limited number of steps were required but foundational mathematical skills were not up to the required standard. The examiner found serious deficiencies evident in algebra, an inability to factorise correctly was specifically mentioned and he also observed mistakes in arithmetical calculations and signs with unexpected frequency. Questions asked in unfamiliar ways requiring more than routine methods he stated, showed up weaknesses arising from inadequate understanding of mathematical concepts and underdeveloped problem-solving and decision-making skills. Similarly, for the Ordinary Level examination of the same year, the Chief Examiner remarked that it was clear that candidates' strengths lay in the area of '*competent execution of routine procedures in familiar contexts'* (p.31) but he observed significant weakness in questions requiring sound conceptual understanding. This was not just the case with advanced material but even with fundamental concepts and skills.

The CER for the LCE in mathematics was not normally produced on an annual basis. In 2001 the relevant government minister requested a report (State Examinations Commission, 2001) for the Ordinary Level mathematics paper of that year. The reasons given were serious concerns expressed in relation to the unsatisfactory performance of candidates in the LCE at that level. The CER found that the low-performing candidates experienced similar difficulties to those identified in previous years but to a considerably greater extent. He stated that these difficulties were related to:

- *'poor computational skills when negative numbers and fractions are involved;*
- *poor skills of manipulation, especially where indices and surds arise,*
- *poor algebraic skills, particularly basic transpositions, multiplication and factorisation; and*
- *difficulty in solving equations, particularly quadratic equations and those involving fractions.' (State Examinations Commission, 2001, p.15).*

There was little improvement evident in the 2005 CER (State Examination Commission, 2005). At the Higher Level the examiner stated that weaknesses in answering questions were traceable to a low standard of foundational skills in mathematics. For example, deficiencies he observed were evident in algebra, the use of brackets, cancelling in algebraic fractions, trigonometry, inequalities and manipulation of expressions involving indices. He noted that weaknesses continued to stem from inadequate understanding of mathematical concepts and underdeveloped problem-solving and decision-making skills. *Mathematical difficulties* experienced by candidates in the Ordinary Level examination included: fractions both arithmetic and algebraic and difficulties with surds, inequalities and percentages other than routine examples. The examiner specifically mentioned that in relation to average candidates, weaknesses were evident with all but the most basic algebraic manipulations and basic routines in solving equations.

The National Council for Curriculum and Assessment (NCCA) is a statutory council and its brief is to advise the Minister for Education and Skills in Ireland on matters relating to curriculum and assessment for early childhood education and for primary and post-primary schools. The NCCA, as a result of the concerns being widely expressed about the quality of the mathematical skills of students leaving the post-primary education system, produced a discussion paper in 2005 intended as a '*fundamental evaluation of the appropriateness of the mathematics that students engaged with in school and its relevance to their needs'* (NCCA (2005), p.3). The objective of the discussion paper was not necessarily to produce revised syllabi for post-primary mathematics, although this was the subsequent outcome. New syllabi for post-primary mathematics, referred to as *Project Maths*, were introduced to twenty-four schools in 2008 and rolled out to all schools in 2010. The new mathematics syllabi covered five strands and these were phased in, in the following order-*statistics and probability, geometry and trigonometry,* *number, algebra,* and *functions and calculus*. The stated aims of the new syllabi were to equip students with:

- *'the mathematical knowledge, skills and understanding they need to succeed in education, work and daily life;*
- *the skills to use mathematics in context; and to solve problems with a range of real-life applications;*
- *a lifelong enthusiasm for mathematics.' (Jeffes et al., 2013, p.15).*

A considerable number of the students, involved in our research project, would have taken the LC examination in 2014 under the new mathematics curriculum. The SEC (2015) was the first Chief Examiner's report (CER) to consider the results of the new mathematics approach initiated in 2011. Therefore, this report is significant when considering our results. In the opening paragraph the examiner commented on the new syllabi. He stated that whereas there was an increase in the extent of statistics and probability covered in the curricula, other areas such as matrices and vectors were removed along with some calculus. In relation to skills, he stated that *'there was a greater emphasis on problem-solving, as well as on the skills of explanation, justification and communication.'* (SEC, 2015, p.3).

The CER for the 2015 examination papers showed an increase in the number of candidates taking the Higher Level examination. The examiner remarked that this was often attributed to the introduction of *Project Maths* and the addition of extra bonus points awarded for the Higher Level mathematics examination. He emphasized, however, that the increase in numbers taking the Higher Level was a stated aim of the Department of Education and Skills and was therefore not solely attributed to the extra bonus points. The Chief Examiner noted that the performance of some Higher Level examination candidates, with respect to their ability to apply basic skills appropriately and accurately, was a cause for concern and that the proportion of the candidature for whom this was a significant difficulty had increased since 2011 with a significant minority of candidates struggling to complete multi-step procedures accurately. At Ordinary Level, as might be expected with the decrease in numbers, the examiner found a lowering of the number of higher achieving candidates. More significantly, he remarked that many candidates displayed a lack of knowledge of standard procedures and a lack of basic competence in algebra and in algebraic manipulation.

Jeffes et al., (2013) produced an interim report investigating the mathematical competencies of second-level students after the introduction of *Project Maths*. The authors compared students in phase one schools (which introduced the revised mathematics syllabuses in September 2008) and students from non-phase one schools which introduced the revised mathematics syllabuses in September 2010. Performance, at this stage of the implementation and across the five strands of the new curriculum, indicated no differences in the students' mathematical achievement. The authors noted that students performed best in Strand 1 (Statistics and Probability) and least effectively in Strand 4 (Algebra) and Strand 5 (Functions). The authors also suggested there was some evidence of positive impacts on students' experiences of, and attitudes towards mathematics and in some instances students appeared to be effectively drawing together their mathematical knowledge across various topics.

Later reports such as Treacy and Faulkner (2015) and Prendergast and Treacy (2015), examining the results of incoming university students' annual diagnostic tests, suggested there was a decline in performance of the basic mathematical skills required for students studying in higher education and showed this decline was particularly significant after the implementation of *Project Maths*.

Prendergast, Faulkner, Breen, & Carr, (2017) asked lecturers to compare the mathematical performance of students educated in the traditional methods with those in *Project Maths*. These comparisons were based on the five strands of the *Project Maths* curriculum. The lecturers' comparison for the strand *statistics and probability* showed 43% were unsure of changes with 11% stating students were much better and 10% that they were better. This was the singular strand where lecturers noted students' performance was much better. More than half the lecturers (58%) stated they were unsure of any changes in *geometry and trigonometry* with 16% noting the students were better. The only strands indicated by the lecturers as worse or much worse were *number* (7%,12%), *algebra* (9%,12%) and *functions and calculus* (21%,14%). The authors noted that the order of the introduction of the strands in *Project Maths* may have influenced these results. Perhaps, the coverage of the strands in a lecturer's module material may also have been a factor.

Prendergast and Faulkner (2018), also comparing diagnostic tests on incoming students before and after the introduction of *Project Maths,* focused on the strand *algebra*. The authors noted that the introduction of the new curriculum coincided with a decline in students' technical algebraic skills. They stated that interviews with classroom teachers showed that this was probably due to a mismatch between the intended implementation of the curriculum and the actual implementation by teachers in the classroom. It is still early days in the implementation of *Project Maths* and perhaps as this research suggested, teachers may still be having difficulty introducing the new curriculum. To comprehensively understand the effects of the new approach to mathematics learning at post-primary level and the subsequent results experienced at third-level in Ireland, further long-term research will be required.

2.3 Lecturer and student perceptions of students' mathematical difficulties

2.3.1 Perceptions reported in the UK

A number of papers examined differences in lecturers' and students' perceptions of the *mathematical difficulties* experienced by students. Perkin, Pell, & Croft, (2007) considered this, examining feedback forms issued to lecturers, Mathematics Support Centres' (MSC) tutors and students attending a mathematics support centre. Where questions, selected from a larger investigation, related to the lecturers' knowledge of students' *mathematical difficulties*, staff answers were categorised into those that taught in the MSC and those that did not. What was significantly different was the percentage of MSC staff, (over 80%), who stated that students found basic manipulation difficult. For other staff, the figure was 30% whereas it was 14% for students.

The only topic, that students perceived more difficulty with, than staff, was statistics. Thirty percent of students, stated that they found difficulty in this area compared to approximately 5% and 16% respectively, for staff and MSC tutors.

2.3.2 Perceptions reported in Ireland

Ní Fhloinn (2009b) described how first-year service-mathematics students completed an anonymous questionnaire with regard to their attitudes and opinions of the mathematics support centre. Students drop-in sessions during semester, specialised drop-in before exams, refresher sessions, revision classes and online resources were rated very highly by the students, with 70-80% of responses categorised as "very good" or "good". When asked for the specific aspects of the centre that they found useful, the one-to-one support (49%) was given as the most useful with tutors (23%) mentioned in responses to approximately half this number.

A project was developed to explore the issues around diagnostic testing and follow-up support for incoming students in a College of Technology in Dublin, (Sheridan, 2013). First year Science students were tested and those who failed to achieve a pass mark of 50% were offered support. The author stated that algebra and arithmetic were the two main areas of difficulty indicated by the diagnostic tests and the author suggested that knowing this in advance allowed her, as lecturer, to change her style of teaching and allow increased time when covering topics needing knowledge of these areas.

Ní Fhloinn, Fitzmaurice, Mac an Bhaird, and O’Sullivan (2014) conducted a large-scale nationwide survey, with first-year service mathematics students in nine higher education institutes in Ireland. This survey explored students’ perceptions of the impact of mathematics support upon their retention, mathematical confidence, examination performance and overall ability to cope with the mathematical demands they encountered at third level. Students were very positive about the effectiveness of mathematics support in all of these areas.

A similar study to that of Perkin, Pell and Croft (2007), in the UK, was that of Ní Shé, Mac an Bhaird, Ní Fhloinn, and O'Shea, (2017). These authors conducted two surveys in Spring 2015. The first was a survey of students enrolled in first year undergraduate non-specialist mathematics modules in four HEIs. A total of 460 students completed this survey. The survey aimed to identify the mathematical topics which students in these modules, determined as problematic and to detect if concepts or procedures caused the greater difficulty. The second survey, seeking similar information by means of a Google form, was emailed to all lecturers teaching Level 1 undergraduate mathematics in Ireland. Thirty-two responses were received. None of the lecturers in the HEIs involved in either the project team (nine in total) or those in the pilot study completed the lecturers’ questionnaire.

The students were asked to rate their ability to answer forty-six mathematical questions and to answer seven open-ended questions. In the open-ended questions the students were asked to indicate if it was the ideas or methods that caused difficulty. The lecturers' questionnaire, had ten open-ended questions. The variation in location of the two surveys, one with students in the four HEIs involved in the study and the other with lecturers from HEIs excluding those involved in the study, may have limited us drawing conclusions from these comparisons. Furthermore, although the lecturers were all teaching first year modules it is not stated if these were the same modules as covered in the student survey and therefore the extent and topics covered may have varied. As a result, differences in terminology may have hindered analysis. An example of this was seen, where it was stated that a small number of students reported matrices as difficult whereas lecturers referred to students having difficulty with linear algebra.

Approximately two-thirds of the students surveyed (Ní Shé, Mac an Bhaird, Ní Fhloinn, & O'Shea, 2017) had taken the Higher Level LC examination and one-third the Ordinary Level LC examination. Students who came to third-level having sat the Higher Level LC mathematics examination were more likely to mention integration as a problem whereas the Ordinary Level students stated they found logs difficult. In answers to the open-ended question in the student survey – '*What topics in first year caused you most difficulty? . . . Please indicate whether it was the methods or the ideas involved that made the topic difficult for you.' — the* students listed integration, differentiation, functions, logs and limits as difficult and rated their ability to understand higher than their ability to answer questions. In addition, a small number of students reported difficulty with matrices, vectors and algebra. In answer to – '*What topics in first year did you find most easy?' — the students* listed algebra, equations and formula, differentiation and integration, functions and graphs, matrices, complex numbers, logs, statistics and vectors. The authors remarked that most of the topics listed as easy by a number of the students, were listed by others as difficult. In answer to the question — '*What procedures and tasks in the first-year curriculum cause most difficulty for your students*?' − the lecturers' responses included formulae, equations and symbols, fractions, linear algebra, logs and indices, differentiation, integration, functions and graphing, trigonometry, probability and statistics and geometry. Eight percent of students referred to finding topics easy or difficult depending on whether they had covered them before whereas lecturers found that students' difficulties with more advanced topics resulted from a lack of basic skills.

A recent study by Duggan, Cowan and Cantley (2018) conducted a series of semi-structured interviews with lecturers teaching first year mathematics across a variety of academic disciplines. This was part of a larger study investigating students' perceptions of their mathematical preparedness for higher education. The lecturers were selected on the basis of their experiences of teaching first year mathematics and were invited via email to participate in face-to-face interviews. In total, nine lecturers agreed to the face-to-face interviews. The interviews investigated lecturers' perceptions of new undergraduates' mathematical skills and also the lecturers' perceptions of the *Project Maths* curriculum.

The authors (Duggan, Cowan & Cantley, 2018) observed several common findings regarding perceptions of *Project Maths* and the '*mathematical preparedness*' of new undergraduates. One of the main issues in the perception of the lecturers was that many new undergraduates lack some very basic concepts and skills, such as algebraic manipulation, fractions and the appropriate use of units. They stated that this finding was a particular concern for lecturers within the STEM disciplines. All of the participating lecturers suggested that new undergraduates had difficulty applying mathematics in unfamiliar contexts and the majority of lecturers in this study suggested that difficulties still existed today in-spite of the introduction of the *Project Maths* curriculum.

2.4 Responses to the mathematics problem

2.4.1 Introduction of mathematics support centres

A major response in both the UK and Ireland to the *mathematics problem* has been the introduction of mathematical and statistical support centres. These centres were most frequently introduced to provide mathematical support to students in the transition from post-primary to Higher education. MSCs were described by Lawson, Croft and Halpin (2003) as '*a facility offered to students (not necessarily of mathematics) which is in addition to their regular programme of teaching, lectures, tutorials, seminars, problem classes, personal tutorials, etc.'* (2003, p.9). The authors also noted that support offered by MSCs could vary significantly but the almost universal aspects were the voluntary nature of attendance and the one-on-one support offered either by drop-in or by appointment. Challis et al., (2004) observed that mathematics support might also include additional support for the curriculum required by weaker or less qualified students during their studies. A guide (MacGillivray, 2008), based on findings from a project investigating the nature and roles of learning support in mathematics and statistics in Australia, was produced to provide information for the university sector on the need for and the provision of mathematics support. This is a quote from the guide.

> *'(A Maths Support Centre) needs sufficient security to attract, train and retain staff, and to play its part in the ongoing and longitudinal data collection and analysis that should be an integral part of its contribution to the university. All universities should ensure that such data collection and analysis are undertaken and performed correctly to provide vital information for university academic management.' (MacGillivray, 2008, p.26).*

2.4.2 Evaluation of mathematics support centres

A short overview of the literature relating to mathematics support centres explaining why and how they developed and their impact on student retention and performance is presented in a paper by Gill, Mac an Bhaird, & Ní Fhloinn, (2010). A more comprehensive review of the literature evaluating mathematics support from the early nineteen-nineties to 2012 is given by Mathews, Croft, Lawson, & Waller, (2012). This sigma report provides evidence of studies evaluating mathematics support from quantitative elements such as prevalence of centres to more complex issues covering evaluation and analysis of the support provided.

2.4.2.1 Prevalence of mathematics support centres

The earliest known data collection in relation to the existence of mathematics support, was that of Beveridge and Bhanot (1994). The rate in the growth of MSCs which has been substantial, especially over the last fifteen years, is an important indicator of success. Lawson, Halpin, & Croft (2001) measured this in the UK by means of an email survey. In a study by Perkin and Croft (2004), 66 out of 106 institutes contacted, stated they had some form of support and the most recent UK study by Perkin, Croft and Lawson (2013) showed a further rise in the numbers to eighty-eight. In Ireland, an audit of provision was undertaken by Gill, O'Donoghue, and Johnson (2008), with the intended purpose of summarising available resources administered by Irish MSCs. Thirteen centres were identified as having mathematics learning support (MLS) and each submitted summary information on the services they provided. The Irish Mathematics Learning Support Network (IMLSN) commissioned a comprehensive audit of the extent and nature of mathematics learning support provision on the whole of the island of Ireland (Cronin, Cole, Clancy, Breen, & O'Sé, 2016). An online survey was sent to thirty-two institutions, including universities, institutes of technology, further education and teacher training colleges with a response received from thirty-one, indicating the presence of support in twenty-seven institutions. This result indicated a significant growth in the number of centres in Ireland since 2008.

2.4.2.2 Qualitative and quantitative data collected

The basic quantitative data collected in a MSC are attendance records. These records were frequently seen as material evidence of the need for support when seeking funding. Mechanisms for collecting usage data varied considerably from the most basic methods where students signed a logbook (Croft, 1997), students signed anonymously with course details including time of entry, and tutor added course and topic of enquiry (MacGillivray, 2009) or where as described by O'Donoghue (2007) students signed a register and data were subsequently added to an Excel sheet. Samuels and Patel (2010) described a Microsoft Access database recording date, time, support given and an interesting addition of a tutor reflection on the assistance they provided. A recent report (Cronin & Meehan, 2015) detailed a sophisticated online system of data collection. For each visit, there was an electronic record of student details, time of visit, the module for which the student was seeking support, along with details, inputted by the MSC tutor and available to the module lecturer, on the exact nature of the *mathematical difficulty* experienced by the student. Data generated over the eight weeks of this research project were obtained from this database.

2.4.2.3 Evaluation of service offered by MSCs

Many studies collected data to measure the impact of the centre on student learning. Gill & O'Donoghue (2007a) examined a number of metrics that might be provided to measure the success of support provision. Diagnostic testing at third-level was frequently described to highlight students at risk of failing examinations. Robinson and Croft (2003) in efforts to improve retention of their engineering students, found that diagnostic testing was a useful tool for identifying students in need of extra support. A number of studies demonstrated positive outcomes when diagnostic test results were combined with follow-on support.

Dowling and Nolan (2006) examined pass rates of at-risk students at Dublin City University (DCU) concluding that their Mathematics Learning Centre (MLC) made a positive contribution to student retention. The importance of collecting data to effectively evaluate such services was highlighted by the theme of the Third Irish Workshop on Maths Learning and Support Centres, (Mac an Bhaird & O' Shea, 2009) which considered the worth of MSCs. The authors discussed the impact of the MSC on the grades of first year students and determined that it had an impact on the majority of students who attended regularly, especially in the case of the most at-risk students. Records of improvement in grade results was also shown by Pell and Croft (2008).

A major report by O'Sullivan, Mac an Bhaird, Fitzmaurice, and N Fhlionn (2014) analysed feedback from over 1,600 first year students taking service mathematics modules at nine higher education institutions across Ireland. The report noted that 22% of the students surveyed had considered dropping out of their courses because of difficulties experienced with the mathematical element of their programme and of these students, 63% indicated that mathematics learning support had been a factor in their decision not to drop out.

Qualitative data collected through the issue of evaluation forms or holding of focus groups were frequently used to provide information on student satisfaction. According to Green and Croft (2012) there were many reasons to collect student feedback and evaluation of this should be considered from the student, support centre, institutional and national perspective. However, they also stated that although it was alien to the support ethos, none the less, it would be like the '*proverbial ostrich,'* (2012, p.3) to ignore the issue of value for money. Student questionnaires were probably the most common method used for detailed evaluation of the centre but as they were frequently only issued to users of the centre, there might be an attendance bias as Lawson, Croft and Halpin (2003) suggested. To overcome this criticism a number of MSCs sent their evaluation forms to all students in the respective first year modules (Croft, 2008; Dowling & Nolan, 2006; Ní Fhloinn, 2009b; Woodhouse, 2004). Croft (2008) warned that it was unusual to receive negative feedback from students so positive results were not surprising but of limited value. Unbiased data obtained from internal or external support (Croft, (2000); Lawson, Halpin, & Croft, (2001); Parsons & Adams, (2005)) were a useful form of evidence to illustrate the success of a centre. Certain restrictions to collecting data were given by Croft (2000). The author suggested that quantifying incidences of students passing their examinations after assistance in the MSC where otherwise they might have failed, is difficult as the cause could have resulted from other avenues of help available to the student. They also advised that diagnostic testing had been found useful to provide evidence of a lack of basic mathematical skills.

2.5 Evidence and feedback of difficulties

Mathews, Croft, Lawson and Waller (2013) in their extensive review of the literature evaluating Mathematics Support Centres, detailed studies relating to feedback on students' *mathematical difficulties.* In this report, they recounted how Beveridge described how feedback from the centre, on difficulties students were encountering, was relayed back to the tutors and quoted Beveridge as follows: '*it was felt much more could have been done to inform mathematics lecturers about common student problems*.' (2013, p.23)

In a comprehensive publication Armstrong and Croft (1999) remarked that a multiple-choice test that was broadly ranging would assess a very limited aspect of any particular topic and as such, low response rates should sound alarm bells but conversely correct responses might not give the full picture. They suggested that, by combining the results of a number of surveys relating to students' confidence in basic mathematics with a subsequent diagnostic test administered to them, it allowed the authors to recommend mathematical areas which were a priority for learning support providers. Amongst their final recommendations was the following:

> *'Further research should be undertaken to identify the sources and validity of perceived problems in teaching, learning and assessment and to develop solutions where necessary.' (Armstong & Croft, 1999, p.71).*

Chapter 3 Methodology

3.1 Introduction

The aim of this chapter is to describe the methods used in the collection and analysis of data on the *mathematical difficulties* as revealed in the *lived experience* of students attending the Maths Support Centre in UCD over a period of eight weeks in Semester 1 2014/2015.

Previously published work in the area of students' *mathematical difficulties* is based around diagnostic testing and supportive follow-up. This is normally carried out prior to third-level instruction with the objective of determining the level of skills known to the students. Diagnostic testing has been shown to be very effective when accompanied by follow-up teaching (Sheridan, 2013; Hodgen, McAlinden and Tomeia, 2014). It also has an important benefit in that it provides information on the whole cohort. In contrast this work setting out to investigate the *lived experience* of an MSC provides a different perspective to previous published work in this area. Although, limited to those who attend the MSC, it provides information not only on the lack of mathematical skills but also on students' *mathematical difficulties* with the module content. It encompasses data on all programmes and levels of students attending the centre. This information, on-going over eight-weeks was instantaneously available online to the module lecturer and lecturers' comments of the usefulness of these data to them are also explored. Prompt support for the student was automatically provided by the tutors in the MSC.

After each student visit to the MSC, the assisting tutor inputs details of the *mathematical difficulties* exhibited by the student into a database. These data, as stated above, are accessible to individual module lecturers in the School of Mathematics and Statistics. Data had been collected on the database for four years prior to the academic year 2013-2014 and the original intention was to analyse these data in order to identify areas of *mathematical difficulty* exhibited by students. The lack of detail in the data, however, meant this was not possible. To obtain in-depth information on the *mathematical difficulties* exhibited by students attending the centre a more comprehensive data collection was undertaken. This collection process, including the coding of the *mathematical difficulties* and the analysis of the data, is summarised in this chapter and more detailed accounts are supplied in Appendix A: Stage 1: Description of the data collection 2009-2010 and Appendix B: Stage 2: Refining codes and working with tutors (Semester 1, 2013-2014)

The initial section of the chapter outlines the background to the project and explains how previous experience in managing the MSC influenced the choice of research questions. Following this, the research questions are presented.

The sequential nature of the data collection and analysis necessitated the presentation of the methodology in four separate stages. The first stage analyses the data collected between the years 2009-2013 and demonstrates the collection's limitation for the research project. The next stage addresses the development of a coding system to allow identification of the *mathematical difficulties* exhibited by the students and the training of tutors to implement a primary coding of these difficulties. This is followed by the third stage, a pilot study undertaken in the second semester 2013-2014 with the objective of testing improvements made in the process of data collection. The final stage involved the eight-week data collection and analysis for the main study in Semester 1, 2014-2015. Analysis and findings for the first three stages are included in this chapter. Findings for the fourth and final stage are presented in Chapter 4.

A series of interviews, 37 in total, with thirteen mathematics lecturers, were conducted by the manager of the UCD MSC, Dr Anthony Cronin, on three separate occasions during the semester of the eight-week data collection period in Semester 1, 2014-15. Dr Cronin agreed, when compiling his interview schedule, to include specific questions to ask the lecturers in relation to this study. Analysis of the lecturers' responses to these questions, are discussed in Section 3.7.

The analysis of a focus group which was held with MSC tutors, in order to gain further understanding of the process of data entry from the tutors' perspective, is presented in Section 3.8.

3.2 Background to research

Explanation for, and solutions to, the *mathematics problem*, where entrants to mathematical programmes at third-level, appeared to lack the quantitative skills needed for university curricula, require serious investigation (Howson et al., 1995). In addition, the increasing importance of quantitative skills across multiple programmes at third-level has been identified by many authors, among them Steen (2001) who states that '*in today's world, the majority of students, who enrol in post-secondary education, study some type of mathematics, tomorrow, virtually all will*' (2001, p.304). Poor mathematical skills across many third-level programmes were evident when one examined the nature of student visits to the MSC in UCD.

Opened in 2004, the UCD MSC is now embedded as a university-wide resource and is funded centrally by the university. UCD is the largest university in Ireland with approximately 25,000 students on campus. Over the three years 2014-2016 the MSC has seen, on average 5,590 visits per annum, with approximately half of these from first year and over a quarter from second year students. This does not fully capture the diverse nature of the visitors. Students seeking support may be enrolled on mathematics and statistics degree programmes; taking mathematics modules as part of another programme of study for example, Agriculture, Business, Engineering or Science; or may be undertaking degree programmes in areas such as Geography, Psychology, Medicine or Social Science, where mathematics modules are not core to the programme, yet often mathematical or statistical knowledge is required for their degree studies.

3.3 Motivation for research

Sensitising concepts have been described by Blumer (1954) as follows: '*whereas definitive concepts provide prescriptions of what to see, sensitizing concepts merely suggest directions along which to look*' (1954, p.7). This proposes the notion of previous experience influencing what is seen by providing initial ideas to pursue and sensitising the researcher to ask particular kinds of questions.

In January 2009, a web application, recording the *mathematical difficulties* exhibited by students at third-level, was designed by UCD MSC to maintain an electronic record of each student visit to the MSC. On this database, for each student visit, the date and length of the visit, the module for which the student required help, their programme of study, and other background information, were recorded. For subsequent visits only the module for which the student sought support required entry. On completion of each session, the MSC tutor added to the database details of the mathematical topic he or she had covered with the student. In addition, the tutor could also add comments on any basic *mathematical difficulties* experienced by the student, if applicable.

Once entered on the database, the module lecturer was then able to access these anonymous *tutor entries* electronically if he or she wished. The feedback to the module lecturer was in the following form:

- *the number of students, from the module, who visited the MSC with a mathematical query;*
- *the length of each visit; and*
- *the nature of each mathematical query.*

In the academic year 2007-2008 the UCD MSC initiated interactive workshops called *Hot Topics*. These are offered to students from a given module, after consultation and input from the module coordinator. A *Hot Topic* is usually organised when a specific mathematical topic is identified by either the MSC or the lecturer as causing particular difficulty for students. The MSC may become aware of the difficulties when a number of students from the module visit the MSC within a short time period. Alternately, the topic may be identified by the lecturer after a quiz or test, or from past experience the lecturer may know in advance that this topic could cause difficulty for weaker students. The aim of a *Hot Topic* is not to cover material from the module, but rather to address gaps in students' pre-requisite knowledge, for example, solving quadratics or applying rules of indices, or to provide additional support in a basic concept or procedure from the module that poses a persistent problem for a small number of students. Due to the interactive nature of the workshops, ideal attendance at a *Hot Topic* is between 10-15 students.

Many MSCs in the UK and Ireland, even where sufficient funding is available, are logistically limited in the support they can provide. Given the high volume and diverse nature of visits to the UCD MSC, it was necessary to critically assess how to maintain a high-quality service in a more efficient way.

Evidence of the mathematical unpreparedness of students for third-level education had emerged through previous studies which focused on the analysis of diagnostic tests as seen in (Croft & Robinson, 2003; Faulkner, Hannigan, & Gill, 2010; Lawson, 2003; Ní Fhloinn, 2009a) or through examination of lecturers' and students' perceptions of these difficulties (Perkin, Pell & Croft, 2007; Ní Shé, Mac an Bhaird, Ní Fhloinn, & O'Shea, 2017). The present research, exploring the *mathematical difficulties* exhibited by students as revealed in the *lived experience* of students attending an MSC over an eight-week period and the reporting of this information back to the module lecturer, provides a different perspective to earlier published work on the subject.

Previous studies examining students' *mathematical difficulties*, from the viewpoint of lecturer or student have found that lecturers' and students' perceptions of these difficulties were in agreement in many cases although, this was not universal on all topics. (Perkin, Pell, & Croft, 2007; Ní Shé, Mac an Bhaird, Ní Fhloinn, & O'Shea, 2017). The importance of a lecturer's awareness of the prior mathematical knowledge of students attending their module, was highlighted in Perkin et al. (2007). Diagnostic testing, usually administered to students entering third-level education, is useful in this respect (LTSN, 2003), particularly, when the ease of administration of these tests is considered. Armstrong and Croft (1999) remarked that diagnostic tests were normally restricted to first year classes with significant mathematical content but that multi-choice diagnostic tests that are wide-ranging, may give limited information on any specific topic. Diagnostic testing although, limited to a number of modules, has the advantage that it provides information on the mathematical knowledge of the full cohort of students taking that module (Sheridan, 2013). The present study, in contrast to this, is limited to students from those modules who chose to attend the MSC for assistance. To maximize the benefits of diagnostic testing, it was recommended that rapid feedback and a structured follow-up process be provided to students (Hodgen, McAlinden & Tomeia, 2014; LTSN,2003). Hodgen et al., (2014) deduced that a referral to a resource was unlikely to be sufficient and that more direct support was required. This study had the advantage in that immediate feedback and assistance was provided to the students by tutors in the MSC. Furthermore, if it is considered that in the academic year 2015-2016, UCD students visited the MSC from over 100 first year modules, 42 of which were not delivered by the School of Mathematics and Statistics, then diagnostic testing, if it was to provide similar information from all modules, would be a difficult and costly procedure.

The initial plan for this research was to analyse the MSC data collected since 2009, in particular, the *tutor entries*. The aim was to identify the most common *mathematical difficulties* experienced by the students across the variety of programmes attending the MSC. The intention, in doing this, was to take an evidenced-based approach to support provision.

By September 2013, more than 16,500 visits to the MSC had been recorded on the database since it was set up in 2009 and Table 3.1 below provides a breakdown, from information, available on the database, of student visits to the MSC from September 2009 until May 2013.

Table 3.1 Breakdown of student visits to the MSC from 2009-2013

Academic Year	**2009-2010**	**2010-2011**	**2011-2012**	**2012-2013**
Number of visits	3,508	4,293	4,401	4,750
Percentage of Level 1 visits	73%	63%	53%	42%
Percentage of Level 2 visits	20%	25%	33%	25%
Percentage of visits from remaining levels	7%	12%	14%	33%

3.4 Research questions

The background and motivation for the study influenced the development of the following research questions.

- *What are the common mathematical difficulties which students present with at the Maths Support Centre from (a) across modules, and (b) within a given module?*

2. *What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?*

3. *In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?*

4. *What feedback, if any, would be most beneficial for lecturers to receive on their students' visits to an MSC?*

3.5 Data collection and analysis

Owing to limitations of the data collected from 2009 to 2013, which are described in greater detail in Appendix A, a more comprehensive data collection was undertaken in order to address the research questions. The iterative nature of the project, where findings in each stage were applied to the data collection and analysis of the following stage, has prescribed the approach in this methodology. Data collection, analysis and findings at each stage of the first three stages of the data collection are described in this chapter. Findings of the last stage is covered in Chapter 4 as these address the research questions. A summary of this can be seen in Table 3.2 below.

Table 3.2 Four stages of data collection and analysis

	Data Collection	Analysis	Results
Stage 1	September 2009-2013 Prior to PhD	September to December 2013	Initial codes Generated
Stage 2	Semester 1 2013-2014 Applying Stage 1 Codes	December 2013 to January 2014	Codes and process refined and tutor feedback incorporated
Stage 3	Semester 2 2013-2014 Pilot study data Collection	May to August 2014	Codes and process refined and tutor feedback incorporated
Stage 4	Semester 1 2014-2015 Main study data collection	From January 2015	Results in Chapter 4

3.5.1 Stage 1: Initial databases (2009 -2013)

The UCD MSC database was initially set up on a MySQL server in 2009 containing a number of fields (or columns).

Among these fields, were two which were populated by the tutors in the MSC. The first was the *mathematical topics* field. This contained a summary of the assistance given to the student and this was populated in the database by the tutor after working with an individual student. The second field was to allow tutors to enter *basic mathematical difficulties* exhibited by the student, if applicable. These columns are referred to respectively, as *mathematical topic* and *basic difficulty* entries, when discussing the data collection up to 2013.

Table 3.3 gives some information downloaded from these fields. Firstly, the total number of entries in the *mathematical topic* column, including the blank entries, in each of the four years is given. The number of entries where the *mathematical topic* entry was blank is shown in the second row. The bottom row counts the total number of entries recorded in the *basic difficulty* column. Further information on this table is available in Appendix A.

Table 3.3 Summary of data count for Stage 1 of research

Entries	**2009-2010**	**2010-2011**	**2011-2012**	**2012-2013**
Number of *mathematical topic* entries	3508	4293	4401	4750
Number of blank entries	80	19	491	482
Number of *basic difficulty* entries	7	8	51	4

On analysis of the data previously collected its limitations became apparent and the focus then became the collection and coding of more detailed data while bearing in mind that this had to be recorded, in a timely manner, by tutors working in a busy MSC. In observing the paucity of both the number of *basic difficulties* entered and the lack of detail in the *mathematical topic* entries*,* the realisation of the importance of requiring tutors to input a single entry rather than two separate entries but more importantly, the need to train tutors to record detailed accounts of the mathematical issues with which students demonstrated difficulty, became the priority.

Firstly, the challenge was to determine what level of detail was required in the data entries and secondly, to work with the MSC tutors to ensure they understood the nature of the data required. Thirdly, ways had to be found such that high quality entries could be recorded and coded as accurately and efficiently as possible in a high-pressure busy MSC. The development of this data collection process is described in detail in Appendix B.

3.5.2 Stage 2: Refining codes and working with tutors (Semester 1, 2013-2014)

To address the research questions, as seen in Section 3.4, it was necessary to develop a process to enable tutors in the MSC to record reliable and detailed data on students' visits in a timely and efficient manner. Tutors in the UCD MSC are mainly PhD students, frequently spend a number of consecutive years working in the centre and are generally excellent and well-experienced teachers.

During the first semester of 2013-2014, with a view to extracting codes (*mathematical difficulties*), using SQL, and realising the difficulties involved when trying to pull out the codes as described in Appendix A, the idea of attaching a specific key to each code developed. Evidence for these experimental codes was subsequently sought from the data collected in semester 1 2013-2014 and this is described in more detail in Appendix B.

The intention was that the tutors would simultaneously record and code, using the respective key, the *mathematical difficulties* experienced by their students. To achieve this, the tutors were required to:

- *Record each of their tutoring sessions in detail, to explain any basic mathematical difficulty that was preventing the student moving forward, and*
- *To simultaneously code the data, where relevant, by adding the appropriate key or keys for each tutor entry.*

What tutors were being asked to do, in real time, was to carry out a primary coding of their entries in such a way that subsequently, each coded area could be extracted by its key for further examination and determination. The commitment and ability of tutors, to record and simultaneously code high quality data, was an essential element in the research process and therefore the next step involved meetings with tutors to communicate and secure their understanding of the proposed method for data coding and collection. These meetings were informal at this stage of the research process.

An example of this type of communication with the tutor is described below. Each entry on the database was available to view and copy in the following form.

Student Number	First Name	Second Name	Programme	Module Covered	Tutor Name	*Tutor Entry*	Date and time-in

When it was necessary to query a *tutor entry* with a tutor, a copy of their respective entry was emailed to the tutor with a request for extra information. For example, the following email, including their copy of the above, was sent to a tutor who had not entered any keys in their *tutor entry*.

> *'I have added another code covering reading data from a graph {g} to the list. So am I correct if I add {g}, {d} to this entry. . . ?'*

It included the view of the tutor's entry on the database. as shown below. Note the *tutor entry* is shown as is the date and time-in but other data have been removed here for the sake of anonymity.

Student Number	First Name	Second Name	DN250	Module covered	Tutor Name	*Interpreting Differentiation* *Graph Reading*	2013-11-14 11:58:54

This is how the tutor replied:

> '{g} should be added. The student was able to find the derivative of a function but not deduce information about the function (that f'(x) < 0 means the function is decreasing, for example). I wasn't sure if the {d} tag should be added but it makes sense if it is.'

In this case, additions, {g} for graphs and {d} for differentiation, would have been added to the *tutor entry* column. These entries were adjusted according to the most suitable codes developed at that time, and the final entry would have read as follows:

'Interpreting differentiation, Graph reading, the student was able to find the derivative of a function but not deduce information about the function (that f'(x) < 0 means the function is decreasing, for example. {g},{d}.'

This entry, as seen above, would then appear if a search, using the respective code key, was made for either coding, that is either *graphs* or *differentiation*.

Here is another example. A tutor was emailed the following extract and asked:

'Was this long division in algebra, the factor theorem or what exactly was the problem? Perhaps I should add {a}?'

Student Number	First Name	Second Name	DN250	Module Covered	Tutor Name	*Factoring cubic equation and polynomial division*	2013-11-14 10:04:27

This is how the tutor replied:

'This was the factor theorem. The student was trying to factor cubic equations and knew how to find the first root/factor, but not what to do then. So I showed her how to use polynomial long division to find the remaining quadratic. You could add {a} (for algebra) here.'

So this was the final entry:

'Factoring cubic equations and polynomial division, the tutor said that this was the factor theorem. The student was trying to factor cubic equations and knew how to find the first root/factor, but not what to do then. So I showed her how to use polynomial long division to find the remaining quadratic. {a}'

At this time, training of the tutors, in accurate coding and detailed data entry, was the main concern. In most cases these changes were made by the researcher. But as tutors became accustomed to the data entry process, queries reduced and many tutors adjusted their own entries to add the extra coding and/or detailed data. However, the responsibility to check the entries always remained with the researcher.

It was while working with the tutors that the requirement for extra codes arose. Tutors were encouraged to contact the researcher if, in their opinion, additional coding of areas of *mathematical difficulty* would be appropriate.

A meeting was held in mid-January 2014 with eight experienced MSC tutors, to inform them of the pilot study which was planned for Semester 2, 2013-14 and to present them with the new list of twenty-three codes with their respective keys (See Appendix B). To further clarify with these tutors, the nature and quality of the *tutor entries* that should be collected, the following two examples were used to describe the difference between a valuable and a less valuable *tutor entry*:

> ***Example A:*** *A student had a problem with limits and continuity and also a problem factoring out 'h' and expanding in a question on first principles {a}, {s} {lim};*
>
> ***Example B:*** *A problem simplifying an expression – common denominator {a};*

where {a} represented an algebraic difficulty and {s} a problem with plus or minus signs. It was explained to the tutors that it was unclear in Example A where the student's difficulty lay. Was it a question of expanding the square or cubic brackets? What was the problem with *limits and continuity*? In Example B the student's difficulty was stated more clearly. The student was unable to simplify the expression using a common denominator.

Suggestions were also sought from the tutors at this meeting on how the efficiency of the data collection might be improved. As a result of the meeting and further discussions, the tutors provided a number of suggestions. Among these were, the introduction of further codes, the use of 'pseudo-LaTex' for entering data and the innovative idea of using carbon-copy notebooks while working with the students. The implementation of these suggestions is discussed in more detail in the next section.

Ethics exemption was sought and received in January 2014 with the stipulation that all students and tutors be informed of their role and that they signify their agreement to act as participants in the research process. Students ticked a box each time they logged into the MSC database if they agreed that the data recorded on their visit to the MSC could be used in the research project. A description of the research project was available online if they wished to view it. Ninety-six percent of students agreed. All data, concerning those students who declined to be part of the research, were removed from the study. Tutors were provided with a description of the proposed research project. All agreed to take part and each tutor signed an individual form indicating that they were happy to be involved in the research. A further requirement was that the online data used for the research would be destroyed at the end of the research project. However, further clarification with the UCD Ethics committee has allowed publication of the anonymised data.

3.5.3 Stage 3: Pilot study (Semester 2, 2013-2014)

Many of the tutor ideas were incorporated in the pilot study. The following extra codes (*mathematical difficulties*) and their respective keys were added on their suggestion.

- *Complex numbers* {cn},
- *Co-ordinate geometry* {cog},
- *Domain and range* {dr},
- *Integration* {int},
- *Partial differentiation* {pd}, and
- *Advanced* {adv}.

A further recommendation, not too widely availed of, was that if a tutor found it beneficial*, tutor entries* could be added using a form of 'pseudo-laTex'. Here is an example of the use of this by a tutor:

> *'Solving complex number equations and expressing complex numbers in polar form. {cn} \frac{z}{1-z}=1-5i; Express z=(1-\sqrt{3}i)^{11} in polar form.'*

The tutors' most innovative suggestion of the use of A4 carbon copy notebooks was also implemented. Each tutor was provided with their own carbon copy A4 notebook. While they worked with a student they used the notebook to record in writing the tuition process covered with the student. The student was given the top sheet and the notebook, containing the copy, was maintained in the MSC. These copies were then checked by the researcher on a daily basis, to cross-check and often further clarify, the tutors' entries. An application of the use of these notebooks and their role in checking the validity of the tutor entries is clearly outlined in Section 3.5.4 below.

Prior to the commencement of the pilot study, full information sheets were issued to all tutors and the data collection was commenced in February 2014. For eight weeks the *tutor entries* were cross-checked on the database by the researcher against the entries in the A4 carbon copy notebooks, sometimes asking tutors for more information if the basic problem was not clearly identified. To provide understanding of the coding process, below are some examples of the data collected during the pilot study:

i. *'Student was finding the critical points of ln(cos(x)) but did not know that if* $\frac{a}{b} = 0$ *then a must be zero and b not equal to zero. {a}, {fr}, {cp}'*
ii. *'How to find a condition that ensures that a 2 x 2 matrix has two equal eigen vectors. Student needed to know that* $b^2 - 4ac = 0$. *{a}, {m}'*
iii. *'Interval notation for open and closed sets, curly bracket set notation means you only have the listed elements in the set, finding the domain and range of a function- emphasis on avoiding negative numbers in square roots and zeros in the denominator for the domain, changing the constant term in a quadratic function shifts the graph up and down the y-axis. {fun}, {g}, {sets}, {a}, {dr}'*

It is important to note at this point that tutors were asked to include any code that they believed might be appropriate. Final coding of the data only took place, after the reason for each code was clarified with the tutor. In the first example above, the tutor confirmed that the student had no problem with finding the critical points and similarly, in the second example student had been able to find the eigen values. Therefore, the final coding of these entries would have been as follows:

i. *'Student was finding the critical points of ln(cos(x)) but did not know that if* $\frac{a}{b} = 0$ *then a must be zero and b not equal to zero. {a}'*
ii. *'How to find a condition that ensures that a 2 x 2 matrix has two equal eigen vectors. Student needed to know that* $b^2 - 4ac = 0$. *{a}.'*

Difficulties indicated by the codes in the third example were confirmed by the tutor and therefore, the coding remained as shown.

Table 3.4 Final coding and corresponding keys

Code	Old Key	New Key
Basic Algebra	{a}	{alg}
Continuous distributions	{stat}	{stat}
Discrete distributions	{p}	{prob}
Converting units	{cu}	{conunits}
Complex numbers	{cn}	{comnum}
Co-ordinate geometry	{cog}	{cogeom}
Critical points	{cp}	{crit}
Differentiation	{d}	{diff}
Domain and range	(dr}	{domran}
Factorisation	{f}	{fact}
Fractions	{fr}	{frac}
Functions	{fun}	{fun}
Graphs	{g}	{g}
Indices	{i}	{ind}
Inequalities	{in}	{ineq}
Integration	{int}	{int}
Limits and continuity	{lim}	{limcon}
Logs	{l}	{log}}
Mathematical expressions	{me}	{mexp}
Matrices	{m}	{mat}
Pattern spotting	{pspot}	{pspot}
Partial differentiation	{pd}	{pardiff}
Sets	{sets}	{sets}
Sign Rules (+/-)	{s}	{sign}
Simultaneous equations	{se}	{simeq}
Trigonometry	{t}	{trig}
Vectors	{v}	{vec}
Modelling	{ }	{mod}
Advanced	{ }	{advl}

The original code *word problems* represented the difficulty students experienced in translating problems from English sentences into mathematical equations. *Modelling*, using the key {mod}, was a more familiar concept for tutors and therefore a more suitable code name and this was the code and key used in the final data collection.

A problem experienced by the tutors during the pilot study was their difficulty in remembering the keys for the respective codes. The full list of codes and the old keys employed in the pilot study and their respective new keys is presented in Table 3.4 above.

The data collection process is described in Section 3.5.4 and the analysis of the data is covered in Section 3.6.

3.5.4 Stage 4: Final data collection (Semester 1 2014-2015)

Data collection for the main study took place over a period of eight weeks, commencing the day of opening of the MSC in September 2014 and ending before the beginning of study week which was immediately followed by two examination weeks. Part of the reason for this timing related to the numbers of students attending the centre, too few and the records would be limited, too many and detailed recording would be difficult. The centre opened in the third week of the semester by which time many students would be seeking help. Attendance would be high for study and examination weeks but also the nature of student visits, for these weeks would be quite different to visits during the semester and mainly related to questions from past examination papers, so these weeks were excluded.

In order to oversee the work of the tutors, the researcher attended the MSC daily over this eight week period. Each evening when the MSC closed, all the tutors' A4 notebooks from that day were collected and each *tutor entry* was cross-checked on the database with the A4 entries in the tutor's carbon copy workbook. The workbooks were returned to the tutors the following morning. There were, on average, 50 *tutor entries* per day. When further clarification, such as coding of a *tutor entry*, was required, sometimes a note was added to the notebook and the relevant tutor was contacted, in person or in other cases by email, as described earlier. It is important to note that *tutor entries,* including coding, were then adjusted where necessary.

To demonstrate this method of validation of *tutor entries,* the following is an example of one database entry and the corresponding entry in the tutor A4 notebook. This is the original *tutor entry*:

> *'Surveyors are looking at a clifftop. They look up at an angle of 24 degrees and move 1500m closer and are at 29 degrees to the top. Find the height?*
>
> *Used method of calling the unknown length x and dividing into two triangles and making two sim, equations and solving for x and height. {trig},{frac},{fact}.'*

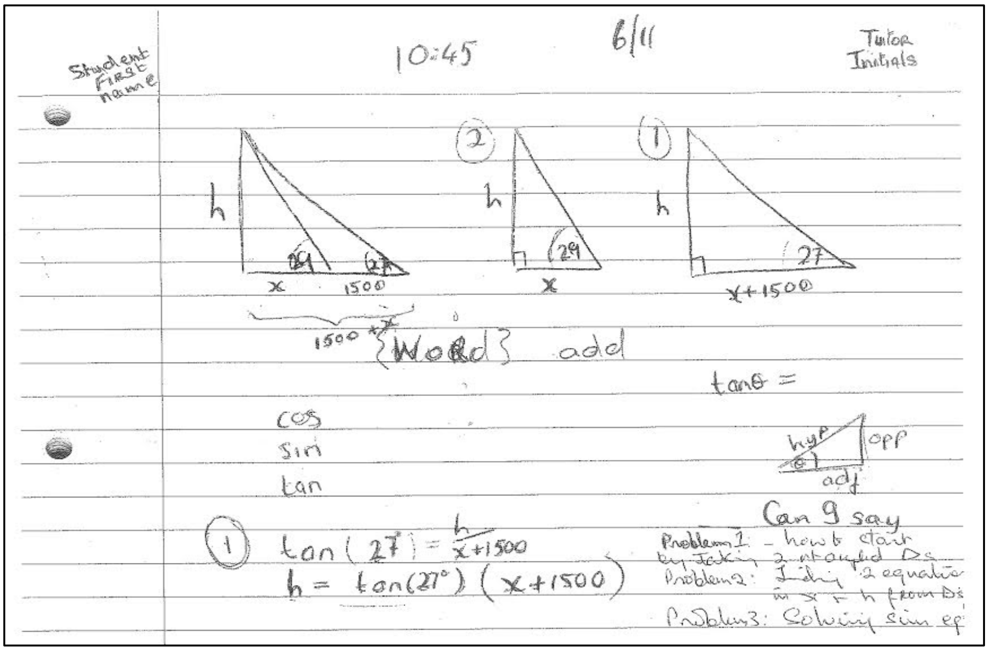
In this *tutor entry*, firstly, the tutor has entered the question for which the student sought assistance. This is shown in the first paragraph. The tutor has then given a description of how he or she helped the student, including the coding. The approach the tutor adopted was as follows: he split the given diagram into two right-angled triangles and found the height. Below is the corresponding entry from the tutor carbon-copy notebook. At the top of the page the tutor entered the first name of the student, the time, the date, and the tutor initials. (The student's actual name has been replaced with 'Student first name' and the tutor's actual initials with 'Tutor

Initials' to preserve the anonymity of both). With this information it was possible to match the page with the *tutor entry* in the database. On the lower right-hand side you can see a hand written query, which was added to the tutor notebook, asking for further explanation of the *tutor entry*. The query was as follows:

Are the following the 'trouble-spots' for the student;

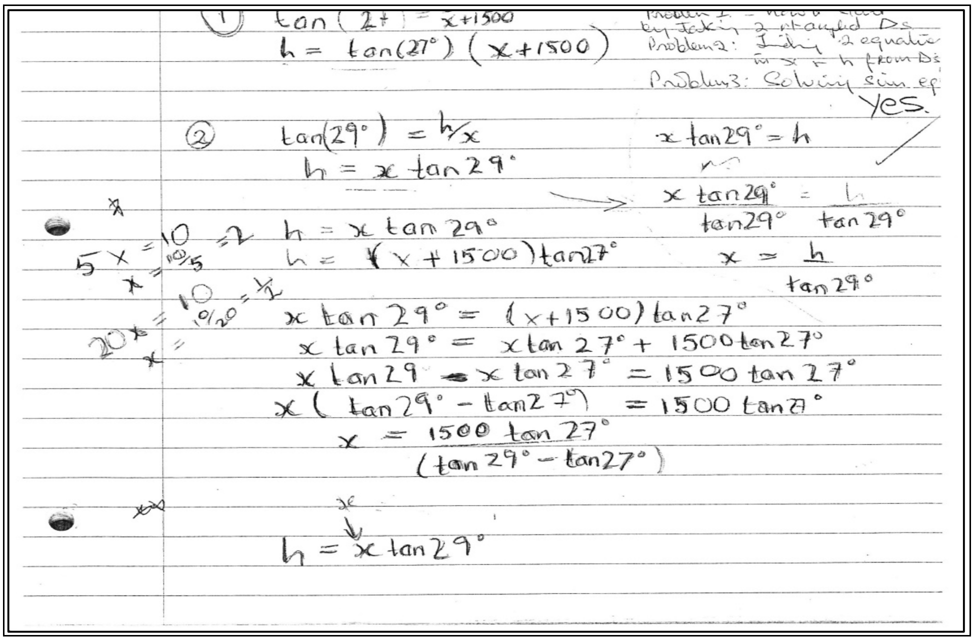
- *how to start by taking two right angle triangles;*
- *finding two equations in x and h from the triangles;*
- *solving simultaneous equations?*

Figure 3.1 Upper half of a page in tutor's A4 work sheet



On the lower half of the same page (see below) you will see the tutor has written '*Yes'* in agreement with the researcher's questions.

Figure 3.2 Lower half of same page in tutor's work sheet



Also note that on the left hand side of the bottom half of the page as seen in Figure 3.2 above, the tutor has written:

$$5x = 10,\ x = \frac{10}{5} = 2; \qquad 20x = 10,\ x = \frac{10}{20} = \frac{1}{2}\ .$$

This suggests that the student experienced difficulty with solving the equation for x, and the tutor demonstrates the method by employing this simple example by way of explanation.

All the extra information was added to the database as shown within brackets. The following is the final adjusted entry in the database.

> *'Surveyors are looking at a clifftop. They look up at an angle of 24 degrees and move 1500m closer and are at 29 degrees to the top. Find the height? Used method of calling the unknown length x and dividing into two triangles and making two sim, equations and solving for x and height. {trig},{frac},{simeq}, {alg}*

Tutor said student did not know how to start by taking two rt angled triangles. Could not find two equations in h and x, had a problem solving the sim. equations. Tan 27 = h/(x+1500) and Tan 29 = h/x. At the side of the workbook the tutor wrote 5x=10, x=10/5 and 20x=10, x=10/20 in explanation while solving the simultaneous equations.'

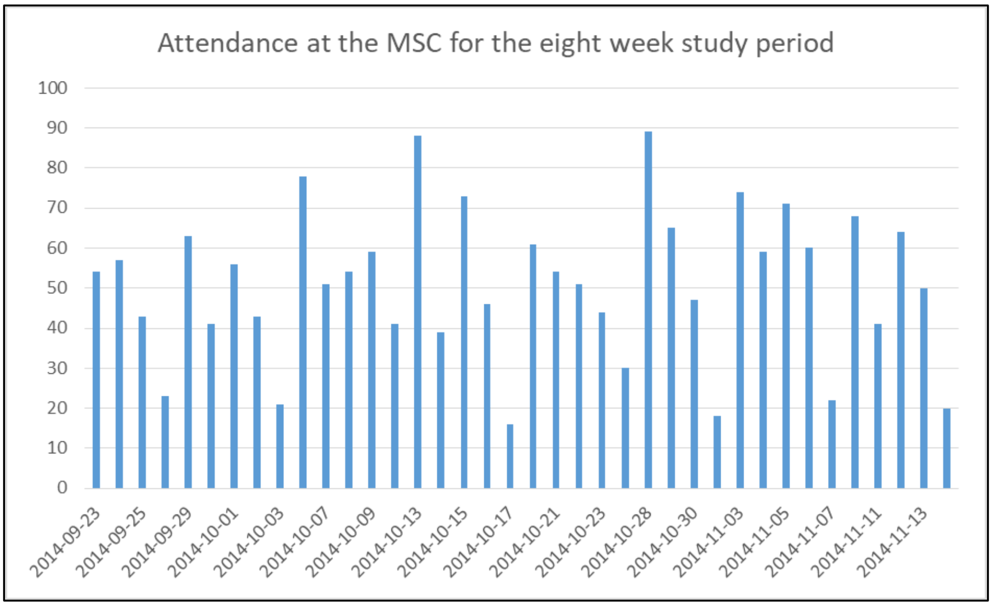
In this manner, on average 50 *tutor entries* per day were cross- checked, over the period of eight weeks of the data collection. This was to ensure that the data recordings included sufficient detail and also, to ensure consistency in the coding process performed by the tutors. On completion of the data collection period there were over 2000 visits by almost 700 individual students studying more than 100 different modules.

3.6 Further data analysis

In January 2015, a preliminary analysis of the data was begun. In total, there were 2,012 *tutor entries* collected over the eight-week period. Each *tutor* entry represented a single visit by a student to the MSC. The number of daily visits to the MSC during the eight weeks of the study period is shown in Figure 3.3 below. Visits for only 38 days are displayed as the MSC opened on a Tuesday so only four days in the first week and a Bank Holiday Monday fell on the 27th October.

Seventy of these *tutor entries,* approximately four percent, were deleted as the students had not given permission for their use. Also, a number of students attend the MSC to work on mathematics but do not actually seek help from a tutor. These visits result in a *blank entry*. Another cause of entries being left blank was that during very busy times, such as coming up to the middle of the semester when many lecturers hold class tests, the tutors may not have the time to enter data. Finally, if a student visits with a very brief query, the tutor may not enter details of this visit on the system. The total number of blank entries was 418. When these were removed from the dataset along with the entries where permission had not been granted by the students, 1,524 *tutor entries* remained. Of these, each had been assigned one or more codes during the data collection process. Some entries did not give information on any difficulty and these were recoded as {nc}. For example, a tutor might say 'student will return when the statistics tutor is on duty.'

Figure 3.3 Number of tutor entries for each day of the research period



3.7 First Research Question

Recall the first research question:

> What are the common mathematical difficulties which students present with at the Maths Support Centre from (a) across modules, and (b) within a given module?

To address this, firstly an explanation of tutor entries, coding, and associated keys is given below. The information in the square brackets in a tutor entry, as stated previously, indicates additional explanations entered by the researcher during the data collection process, after consultation with the relevant tutor. This is evident in the example below. It is coded as a difficulty with indices, hence the key {ind} and the extra information is added by the researcher:

> *'Student came in with a problem with indices. The student was confused between 2^{1/3} and 2^{-3}. The student thought that 2^{1/3}=1/(2^3) but was fine once it was explained. [tutor wrote cube root (2) = 2^(1/3); 2^(-3) = 1/2^3.] {ind}'*

The following are five adjustments made to the coding conducted in the eight weeks of the data collection:

1. Tutors entered any code that, in their opinion, represented the difficulties exhibited by the student. For example, an entry, coded as factorisation {fact}, may also be coded as basic algebra {alg}. The same might be true of an entry coded as simultaneous equations {simeq} or fractions {frac} – each of these could also be coded as basic algebra {alg}. Thus basic algebra was in essence an umbrella term or category in addition to being a rather large, catch-all code for algebra. To adjust for this, all entries coded under basic algebra but also coded elsewhere, were removed from the code basic algebra.

2. *Differentiation* {diff} was another umbrella term or category, in that it included the code *critical points* {crit}. These entries were also removed from the code of *differentiation*. All entries, where a student sought help with *critical points,* were entered either under the code *critical points* {crit} in the case of single-variable functions or under *partial differentiation* {pardiff} for multi-variable functions.

The reason for this distinction was that critical points for single variables are covered at second-level but for multi-variables it is introduced for the first time at third-level in UCD.

An example of the application of the first and second changes can be seen here:

> *'Student was having difficulty finding critical points of a function. Problem was with factorizing the equation and then finding the zeros as the lecture notes jumped from differentiating the function to the final answer.[$f'(x) = 6x^2 + 6x - 36 = 0$)] {alg}, {fact}, {diff}, {crit}.'*

It was originally entered, as shown above, with keys to four codes. It was coded as *factorisation* and *basic algebra* and also as *differentiation* and *critical points*. In the adjustment process it was re-coded with the keys {crit} and {fact} remaining and the keys {diff} and {alg} were removed as seen below. The reason for {crit} remaining was that on further discussion with tutor, the tutor stated that student had not realized critical points were given in the form (x, f(x)) and could not explain how the y value was calculated in the answer the lecturer had provided:

> *'Student was having difficulty finding critical points of a function. Problem was with factorizing the equation and then finding the zeros as the lecture notes jumped from differentiating the function to the final answer.[NC $f'(x) = 6x^2 + 6x - 36 = 0$ $student\ also\ had\ problem\ finding\ f(x)\ for\ crit\ point\ (x, f(x))$ NC] {crit}, {fact}.'*

3. *Modelling* had been a catch-all code in a different sense to those described above. Extra *mathematical difficulties* were coded within *modelling* for which no codes had been

provided. Two new codes were therefore introduced to cater for these topics. *Modelling* was then recoded under three separate codes. These were *discrete mathematics {*disc*},* *mechanics {*mech*}* and *modelling* {mod}. Differential equations did not have a separate code but were coded incorrectly under the code of *differentiation* {diff}. These entries were removed from *differentiation* and placed under *mechanics*. *Sets* might have been coded under *discrete mathematics;* however, as it had been given a code previously it was not included in the new code but remained as a separate code.

4. Some students in Level 3, or higher modules, sought help for more advanced topics such as that shown below for a Level 3 statistics module:

> *'ARIMA Model Time Series {adv}. (An Autoregressive Integrated Moving Average Model).'*

These advanced *tutor entries*, 298 in total, were coded by the tutors as *advanced* and given the corresponding key {adv}. Forty-seven of these entries were incorrectly coded with the key {adv} and were re-coded using the appropriate keys. Another 25 *tutor entries* had not been coded as {adv} and should have been; these have since been recoded with the {adv} code, resulting in a total of 276 *tutor entries* for this code. Possibly tutors' unfamiliarity with the topic and a resulting inability to help the student provides an explanation for these entries being coded incorrectly. Also, in a number of entries for {adv}, only basic *mathematical difficulties* were evident and these were recoded under the various appropriate codes. Leaving a total of 252 entries coded as {adv}.

5. A number of entries are included on the database but are not coded. For example, a lecturer conducted a number of extra tutorials in the MSC, for an Access programme and although these data were entered on the database, *mathematical difficulties* were recorded in relatively few cases and therefore only these specific difficulties are coded. Also, a number of tutor entries are not assigned a code as the descriptive text contained no information relating to any *mathematical difficulties*. For example:

> *'Unable to help, student will return when the statistics tutor is on duty.'*

Any single *tutor entry* may contain a number of codes and, therefore, the total number of coded *mathematical difficulties* may exceed the total number of *tutor entries*. The total number of *mathematical difficulties* was 1800 and the total number of coded *tutor entries* was 1,320. Table 3.8 gives the final list of 31 distinct codes that were used in the analysis of the data and displays the number of *mathematical difficulties* in each.

Table 3.5 Codes with the number of mathematical difficulties in each code

Code	**Number of *Mathematical Difficulties***
Advanced	252
Vectors	142
Discrete mathematics	142
Matrices	124
Mechanics	108
Continuous distributions	108
Basic algebra	89
Differentiation	71
Indices	65
Integration	64
Graphs	63
Partial differentiation	58
Mathematical expressions	57
Functions	48
Limits and continuity	47
Trigonometry	45
Logs	43
Modelling	39
Discrete distributions	36
Complex numbers	34
Fractions	26
Factorisation	24
Sets	23
Critical points	22
Sign rules	18
Inequalities	16
Domain and range	11
Co-ordinate geometry	10
Sim. equations	7
Converting units	6
Pattern spotting	2

It was evident that a number of these codes could be described as belonging to definite areas of mathematics such as algebra, calculus, statistics, or applied mathematics. Whereas, a code such as, *advanced,* was quite distinct as it was both high in the number of difficulties recorded and also an amalgam of various mathematical topics. Finally, there were a number of codes which did not fall naturally into any particular area and these might be classified under a separate heading.

Therefore, it was decided that the allocation of codes to a number of groups would be the optimum approach to illustrate the research findings. To present the results it was decided to categorise each code under a relevant group with the individual code *advanced* allocated to a single group. These groups with their individual codes are displayed below:

- ***Algebra**: matrices, discrete mathematics, basic algebra, indices, factorisation, complex number, logs, fractions, inequalities, sign rules, and simultaneous equations.*
- ***Calculus**: differentiation, integration, graphs, functions, partial differentiation, limits and continuity, critical points, and domain and range.*
- ***Applied Mathematics**: vectors, mechanics, and trigonometry.*
- ***Statistics**: discrete distributions, and continuous distributions.*
- *Advanced*
- ***Other Codes**: mathematical expressions, sets, modelling, co-ordinate geometry, converting units, and pattern spotting.*

Sets might have been included under *discrete mathematics* however, as it had been allocated a separate code and the numbers in the code were small it was included in *other codes.*

Further description of the contents of each of the six groups including a number of examples will be presented in the findings in Chapter 4.

3.8 Second and third research questions

Recall the second and third research questions:

> 2. *What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?*
>
> 3. *In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?*

To address these questions, the nature of students' visits was explored and possible benefits for the operation of the MSC highlighted. For each *mathematical difficulty* the module and module level for which the student sought assistance was recorded on the database. The level of the module, associated with each *mathematical difficulty,* was recorded. The distribution of the *mathematical difficulties* from Level 0-4 was examined. For information on UCD level descriptors, please see Appendix D.

The *mathematical difficulties* for Level 0-1 were classified according to the nature of the difficulty. The classifications of *Prior Learning* and *Module Content* indicate whether *mathematical difficulties* were related to knowledge that was a prerequisite to the module, or to the module content itself, respectively. Further explanations of these are detailed in the findings in the next chapter.

Finally, the relationship, between the number of *mathematical difficulties* and the modules from which students attended the MSC, was investigated.

Results from the analysis of these findings will be presented in Chapter 4.

3.9 Analysis of the focus group

A focus group is a qualitative technique that emphasises dynamic group interaction and provides specific information on a selected topic in a relatively short period of time (Vaughn, Schumm, & Sinagub, 1996). It is important that there is homogeneity in the composition of the group to allow for similarity in the background of participants so all members can contribute and are comfortable talking to each other, but variance in the perspective is important in order to generate discussion (Morgan, 1996). A decision to conduct a focus group, with a number of the MSC tutors, was decided to partially address the third research question:

In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?

In June 2015, at the end of Semester 2 following the main data collection, a focus group was conducted with ten of the MSC tutors. Their perspectives in addressing the issues were of particular interest as they had been involved in the data collection process for the study. They were also experienced MSC tutors, two had been tutors in the MSC for four years, one three years, and four were in their second year of tutoring. Associate Professor Maria Meehan agreed to act as moderator. A social desirability bias (sdb) is described by King and Bruner (2000) as a desire of a person to respond to a question in a socially desirable manner and the authors suggest this is a neglected aspect of validity checking. Recognising this possibility, the presence of the researcher at the focus group, although not participating in the discussion, may have introduced an element of sdb.

The focus group schedule including questions and prompts is available in Appendix E).

The participants were asked for their understanding of the type of information a feedback entry should contain and how their understanding of this had developed. They were also asked what, if anything, they found helpful in developing this understanding and whether the process of entering and coding the data had affected their practice in any way? They were queried on their preferences for entering data and what, if any, improvements in the process might be possible.

The focus group which lasted 81 minutes was audio recorded and fully transcribed. Firstly, the researcher listened to the recording and read the transcript a number of times. The transcript was then analysed to address the following questions:

- *What was the tutors' understanding of the purpose of a tutor entry?*
- *What, if any, improvements to the data entry process were identified and how might these be achieved?*

Findings of the focus group are reported in Chapter 4.

3.10 Analysis of Lecturers' Interviews

To address the fourth research question:

4. What *feedback, if any, would be most beneficial for lecturers to receive on their students' visits to an MSC?*

The lecturers' opinions on the suitability of the feedback provided by the *tutor entries,* were sought. Dr Anthony Cronin, the manager of the UCD MSC conducted a series of interviews, on three separate occasions, in semester 1 2015, as part of a larger project on how lecturers with large first year modules receive feedback on their teaching. Interview 1 was a warm up meeting conducted in week 4 of the semester and mainly for the purpose of explaining the aims of the research project and checking that the lecturers were able to access the Maths Support Centre (MSC) feedback data for their module. Some lecturers interviewed were assisted by the interviewer in gaining access to their module feedback. Possibly, a number of these lecturers had not accessed the feedback previously. The number of entries in most cases were limited at this time, as they were conducted in the first week that the MSC had opened. Interview 2 was conducted in week 8 when there were significantly more feedback entries available for the lecturers to read through and comment on. The final interviews were conducted in week 15 of the semester, three weeks after lectures were finished and therefore allowed more time for deeper discussion with lecturers on the quality and suitability of the feedback. When compiling his interview schedule, Dr Cronin agreed to include a number of open-ended questions that were appropriate to this study. The lecturers' comments in relation to the *tutor entries* were of particular interest since Dr Cronin's interviews were conducted at the same time as the data collection for this study. The lecturers had access to the *tutor entries* from this research study and this meant they were well-placed to comment on the validity, or otherwise, of these entries.

These face-to-face, semi-structured interviews with thirteen lecturers took place, as stated above, in weeks 4 and 8 of semester 1, 2014-2015 with the third interview shortly after the end of the semester. The lecturers interviewed all taught Level 1 or Level 2 mathematics modules and the *tutor entries* were available to them. These modules varied and included modules taught to mathematics majors, engineering students and students taking a mathematics module as part of a non-mathematics degree programme. All interviews were audio recorded and the second and third interviews were transcribed.

During each interview, the manager presented the lecturer with the relevant *tutor entries* for his/her module up to that time in the semester, and asked the lecturer to read and comment on the data. Specifically, each lecturer was asked the following questions:

- *Do these entries make sense to you – in other words, are the entries detailed enough for you to recognize the difficulty that the student is having?*
- *When reading through the feedback comments can you identify whether the comment relates to (a) specific module content or (b) some basic maths/prerequisites that the student is struggling with or (c) is it something else?*

- *Do you find the Maths Support Centre feedback on your module useful?*

The lecturers were informed that the detailed entry of data as undertaken over the eight weeks of the research data would cease when the research was complete. At the interview conducted in the middle of the semester, each lecturer, when presented with the *tutor entries,* was asked if he felt the extra comments, added by the researcher to the *tutor entries* during the eight weeks of data collection and highlighted in blue by the manager, were superfluous.

Firstly, the researcher listened to the interviews and then read the transcript a number of times, to explore answers to the above questions. The relevant parts of the interview transcript was then analysed in order to address the following themes:

- *Did the lecturers recognize the tutor entries as arising from their module?*
- *Were there instances, when the lecturer felt that the feedback related to a student's lack of pre-requisite knowledge for the module?*
- *Was the feedback from the MSC useful to the lecturer? If so, in what ways?*
- *Was the level of detail suitable – too much/too little?*
- Did the opinion of lecturers vary between interviews?
- Were there changes in lecturer practice as a result of learning from MSC feedback?

Results of these findings are presented in Chapter 4.

3.11 Issues Arising and Validation of the Data

A problem that arose during the analysis of the data was that a number of the recorded tutor entries, did not relate to the module that the student was seeking help for. The module code, at that time, was entered by the student when he or she logged in at the start of each visit, and quite often, the module code was entered incorrectly. Significant improvements had been made to the online data system by the manager Dr Cronin in January 2015 and this allowed access to the background information on all students attending the MSC (Cronin & Meehan, 2015). This meant all module entries could now be validated and altered on the database, by the researcher, to show the correct module. This delayed the analysis by at least two months.

Chapter 4 Results

4.1 Introduction

Many universities issue a mathematical diagnostic test early in the first semester to incoming Level 1 students. These can be beneficial in revealing the deficits in prior knowledge and basic skills of the incoming cohort. In contrast, the data gathered in this research comes from the *lived experience* of students attending a mathematics support centre over an eight-week period in the first semester 2014/2015 and although limited to those students who seek help in the MSC, aligns more specifically with the module content.

Results of diagnostic testing based on the whole cohort being examined may show that students across the cohort are not good with inequalities or fractions, but data recorded from the *lived experience* showed very few people came for help with these topics perhaps, because they are not part of their module content. Data from the *lived experience* were limited to those students who chose to visit the MSC. This could have been caused by a range of factors such as the lecturer being conscientious and promoting the MSC to students, the lecturer being very poor and the students coming to the centre out of desperation, students motivated by upcoming worksheets or examinations, or perhaps, students seeking the highest grades.

The results are presented in four sections in this chapter. The first section details findings from the analysis of the *mathematical difficulties* exhibited by students attending the

Maths Support Centre (MSC) during the eight-week research period and addresses the following research question:

> *What are the common mathematical difficulties which students present with at the Maths Support Centre from (a) across modules, and (b) within a given module?*

The second section examines the general nature of students' *mathematical difficulties* observed in the MSC. Specifically, it examines whether students' *mathematical difficulties* relate to an issue with prerequisite knowledge for the module for which they are seeking help, or relate directly to the module content.

> *What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?*

The prevalence of *mathematical difficulties,* in modules where student attendance was high, is investigated. Information derived from a focus group with MSC tutors the aim of which was to improve the feedback process to lecturers is also considered. In exploring these findings, the following research question is addressed:

> *In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?*

The final section detailing findings from analysis of interviews with lecturers, as described in the previous chapter, will address the fourth research question:

> *What feedback, if any, would be most beneficial for lecturers to receive on their students' visits to an MSC?*

In the following sections, when giving an example of a *tutor entry* it will be presented in the form that it was inputted by the tutor and also include the extra information added by the researcher as described in the final paragraph of Section 3.5. A *tutor entry* may be coded to more than one *mathematical difficulty*. Thus, when describing a particular code, just the extract of a *tutor entry* that relates to the code in question may be presented.

4.2 Common mathematical difficulties

The results in this section address the following research question:

> *What are the common mathematical difficulties which students present with at the Maths Support Centre from (a) across modules, and (b) within a given module?*

The total number of visits to the MSC over the eight-week period was 1942 visits excluding 70 that were eliminated as consent had been withheld by the students. A number, 418 in total, with blank entries and 221 with insufficient information were also removed. This left a balance of 1303 visits suitable for analysis. Each of these visits is represented by a *tutor entry*.

Mathematical difficulties evident in these *tutor entries* were examined under thirty-one topics referred to as codes. In addressing the research questions each of the *codes* has been classified as a member of one of the following six groupings:

- Algebra;
- Calculus;
- Applied Mathematics;
- Statistics;
- Advanced;
- Other.

Examples of difficulties exhibited in various modules will be presented in the following sections. *Tutor entries,* throughout, are displayed shaded and in italics. Any single *tutor entry* may contain a number of codes and, therefore, the total number of *mathematical difficulties or codes at 1800* exceeds the total number of *tutor entries* or visits at 1303 as already described above. The term *mathematical difficulties* will be used when the results are presented unless stated otherwise. Where *mathematical difficulties,* in any grouping, number less than thirty for a given code they will be placed in an appendix. See Appendix C for further details of these codes.

Minimum entry levels in mathematics are required for almost all programmes in UCD. The mathematics curriculum for the Higher and Ordinary Level Leaving Certificate mathematics examination has been adopted in this research as a measure of the minimum pre-requisite knowledge of mathematics required for modules taught in these programmes. The levels are based on grades achieved in the Leaving Certificate examination. Higher Level marks are indicated by a 'H' placed in front of the grade and Ordinary Level by an 'O'. For example, H3 indicates the minimum requirement is a grade 3 in the Higher Level examination and O2 is a grade 2 in the Ordinary Level.

The levels for various programmes are seen in Table 4.1 below.

Table 4.1 The minimum mathematics entry requirements set by UCD

Module Category	**Minimum Level of Mathematics Required**	**Programmes**
Category A	O6	Access, Agricultural Science, Arts and Humanities, Medicine, and Architecture.
Category B	O2	Science, Computer Science, Commerce, and Sports and Exercise Management.
Category C	H4	Mathematical Sciences and Physics, Engineering, Economics and Finance, and Actuarial and Financial Studies which requires a H2

It is important, however, to add that although they constitute the minimum requirements, the recommended levels for some modules within a degree programme may be higher. For example, students entering the Bachelor of Science degree programme must have achieved a minimum O2 in the Leaving Certificate Mathematics examination. However, if they wished to pursue Mathematics or Physics to degree level they were recommended to have a minimum mathematics pre-requisite of a H3. Also, many students may have a higher mathematics entry level than is required for the programme. However, since access to actual Leaving Certificate results for each student was not available to the Maths Support Centre our analysis is based on Leaving Certificate mathematics requirement for the module.

4.2.1 Grouping 1 - Algebra

Eleven of the codes from Table 3.8 were classified under the grouping of *Algebra and the number of mathematical difficulties for each code is shown in Table 4.2 below.*

Table 4.2 Codes and number of mathematical difficulties in Algebra

***Algebra* grouping**	**Number of *Mathematical Difficulties***
Discrete mathematics	142
Matrices	124
Basic algebra	89
Indices	65
Logs	43
Complex numbers	34
Fractions	26
Factorisation	24
Sign rules (+/-)	18
Inequalities	16
Simultaneous equations	7
Total *mathematical difficulties*	588

Each code with greater than 30 *mathematical difficulties* associated with it, is described below with a number of examples of the issues encountered. The remaining codes are described briefly in Appendix C. The *Algebra* grouping represented approximately 33% of all the *mathematical difficulties* recorded over the eight-week research period.

4.2.1.1 Discrete mathematics

The main *mathematical difficulties* coded under *discrete mathematics* were found in the following key areas: proof by induction, combinations and permutations, binomial theorem and coefficients, modular arithmetic, proof by contradiction, inclusion/exclusion principle, lowest common multiple, graph theory, and group theory. Other areas such as injective and surjective functions, convergent and divergent sequences, Euler’s or Wilson’s theorems appeared less frequently. The *mathematical difficulties* coded under *discrete mathematics* were experienced by some students studying a Category B module but mainly by those studying Category C modules. (See Table 4.1). The more persistent difficulties are highlighted in the following examples:

Proof by induction

These difficulties represent 23% of the total difficulties for *discrete mathematics*. Examples included here are issues with understanding the principle of induction, taking the correct initial step, problems completing the $(n+1)^{th}$ step, the final step of the proof, and the use of strong induction. This is an example of a query on induction by a student studying a Category C module:

> *‘Prove for all n: 1 + x + ... + x^{n} = \frac{x^{n+1}}{x-1}. Student had trouble seeing where to apply induction hypothesis in n+1 step.’*

Binomial expansion

The main issue was that students did not know the binomial theorem or if they did were unable to apply it to answer questions. Approximately 15% of *mathematical difficulties* for the code *discrete mathematics* were found in this area. This is where four students in a Category C module came in for assistance with this topic:

> *'Find the coefficient of X^3 given f(x)=(1 + 2x + 2x^2)^5. Told student to take the binomial expansion of (a + b)^5 using a=1 and b=(2x + 2x^2).'*

Modular arithmetic

Students difficulties in these cases related to the inability of students to understand modular arithmetic or answer questions needing this knowledge. Difficulties relating to this area represented 14% of the difficulties for *discrete mathematics*. This difficulty was experienced by a student taking a Category C module.

> Tutor explained Fermat's little theorem and how to apply it, also showed given 2^(46) congruent to x mod(47) to find value of x.

Inclusion/exclusion

In a number of cases, students were attempting to work answers out manually rather than using the principle of inclusion/exclusion. *Mathematical difficulties* with inclusion/exclusion were evident in 13% of difficulties in the code *discrete mathematics.* The following is a difficulty experienced by a student studying a Category C module:

> 'How to calculate phi(1000) using inclusion exclusion. Student had the solutions but needed a Venn diagram to aid understanding.'

Finding lowest common multiple (LCM) and greatest common divisor (GCD)

Mathematical difficulties in this area mainly related to using the algorithm to find the GCD and accounted for 9% of the total difficulties for *discrete mathematics.* A student studying a Category C Module presented with the following:

> *'Student needed to understand how to find the gcd of 12345 and 67890 and also find s and t* where d =sa +tb.'

Proof by contradiction

Understanding the principle of proof by contradiction was the issue in these *mathematical difficulties* and represented approximately 4% of the difficulties for *discrete mathematics.* This is an example from a Category B module:

> *'Queries about proofs, why use contradiction, how to show the square root of a prime is irrational?'*

Other areas of difficulty coded under the *discrete mathematics* were present in small numbers.

4.2.1.2 Matrices

The code of *matrices* included difficulties in the following areas: adding and subtracting matrices, multiplying matrices by scalars, multiplying matrices, solving systems of linear equations using row reduction, writing the solution to a system of equations when there is a free parameter, finding the inverse of a matrix, rules and properties of matrices and determinants, and matrix transformations. A number of examples of these *mathematical difficulties* exhibited by students attending the MSC are given below:

Properties and algebra of matrices

Examples included here are issues with adding matrices, multiplying matrices and applying rules of matrix multiplication. Understanding how a matrix represents a set of linear equations and understanding the meaning of the identity matrix and of an inconsistent matrix were other difficulties experienced by the students. These queries were present in approximately one third of the total *mathematical difficulties* for the code of *matrices*. Below is an example of a query on matrix multiplication by a student studying a Category B module:

> *'Student was unsure of the method of matrix multiplication and the difference between AB and BA and why the order matters.'*

Gaussian elimination calculations and parametric solutions

Assistance sought in connection with the application of matrices to solve equations with parametric solutions accounted for more than 20% of students' *mathematical difficulties* classified in the code of *matrices*, with an additional 20% of *mathematical difficulties* relating to Gaussian elimination with unique solutions. Below is a typical example of difficulties students encountered when finding solutions to parametric equations:

> *'Student didn't know how to start simultaneous equations with two equations but three un-knowns, didn't understand about free variables, [Tutor] went through an example in detail.'*

Understanding and calculating the determinant and inverse of a matrix

This was a difficulty in relation to approximately 24% of *mathematical difficulties* with queries classified as *matrices*. Difficulties in this area were experienced by students from both Category B and Category C modules. (See Table 4*.1).* The following is an example of a query from a student taking a Category C module:

> *'What is the matrix of minors and co-factors and how to calculate them, how to use these to find the inverse of A.'*

Other difficulties

These related mainly to more advanced topics such as using matrices to find cross products, calculating eigenvalues and eigenvectors and proving that a matrix is positive definite. For example this is a calculation query from a student taking a Category C module:

> *'Working on cross product problems, [student was] unsure of how to calculate'.*

4.2.1.3 Basic algebra

Mathematical difficulties are coded as *basic algebra* where they relate to students' understanding and execution of very basic algebraic techniques. The term 'very basic' describes algebraic techniques that the majority of students might be expected to have mastered for the Irish Junior Certificate or the Ordinary Level Leaving Certificate mathematics examinations. The *mathematical difficulties* coded under *basic algebra* were very varied in nature but the more persistent difficulties are summarised in the following examples:

Simplifying or expanding an algebraic expression

This area of difficulty was most commonly found in queries experienced by students attending Category A and B modules. This is an example of such an entry from a Category A module:

> *'Basic algebra, simplifying equations, multiplying equations, gave student some MSC leaflets they wanted more examples to try themselves. Simplifying 7(2x^2 +6x +3) - (6x^2 +10x +6) also (x-5)(x^2 +3x +6).'*

Simplification of square roots of numbers

This difficulty was mainly shown in a core module for students wishing to pursue a degree in *physics* or *applied mathematics* but this module is also open to a student in any Category B or C module. For example, in this case it was a Category B module:

> *'Finally explained direction of a vector by using tan^(-1) (16i -8j) and found unit vector (16/sqrt{320})i - (8/sqrt{320})j , this showed up problem with sqr roots, (Tutor) showed \sqrt{20})=\sqrt{4.5}= = \sqrt{4}.sqrt(5) = 2 \sqrt{5} and same with \sqrt{18} then showed \sqrt{320} =8 \sqrt{5}.'*

Replacing a variable

Students studying both Category A and Category B modules experienced difficulties here. The problem was evident in a Category A module where a student was differentiating a function from first principles and failed to find f(a+h) correctly. In a Category C module, the issue arose in proof by induction problems, for example:

> *'While doing an induction problem they didn't realise that if you replace k with (k + 1) into this equation 2^2k + 3k -1 you will get 2^2(k +1) + 3(k + 1) -1 they said it was 2^2k+1 + 3k - 1 ie they forgot to include the brackets.'*

Following lecturer's notes or answer given for a worksheet question

Eighteen percent of *mathematical difficulties,* with which students presented at the MSC for a difficulty in the area of *Basic Algebra*, were for a Category C calculus course and this example shows a difficulty with their notes experienced by a student studying this module:

> *'Prove that sinh^(-1)x=log(x+sqrt(x^2+1)). Student had trouble following proof in notes. Wasn't sure how to get from line e^y - e^-y=2x to e^2y -1=2xe^y. Wasn't really aware that if you apply a change to one side of equation, must do it to the other side.'*

4.2.1.4 Indices

The majority of entries under this code illustrated students' difficulties in knowing and/or understanding the rules of indices and the application of the rules in different areas of mathematics. Some examples of the difficulties which students presented at the MSC for assistance with this code were as follows:

Indices in non-mathematics modules

Difficulties with scientific notation were shown, for example, by students studying chemistry, biology and physics modules. Students in more advanced level modules, however, also experienced problems as seen in the following example of a *tutor entry* relating to a student difficulty in a Level 3 Economics module:

> *'Was having trouble following an example in the notes, was getting confused with indices and how to solve for n in 5n^-0.5=4.'*

Fractional and negative exponents

A Category B module revises the topic of indices and students are given a worksheet in this area. Fractional and negative exponents were the main areas of difficulty as seen in the example below and this was further confirmed by *mathematical difficulties* for a *Hot Topic* (see Section 3.3) subsequently organised for this module:

> *'Main problem was both negative and nth root powers. (Tutor) gave the students 8^(-4/3) to answer they were fine with it but one student queried if you had an expression 2 x 8^(-4/3) would the 2 also be to the power of 4.'*

Students studying a Category C Calculus module also found difficulties in simplifying exponents. Here is an example of a *mathematical difficulty* where a student looked for help in more basic knowledge when answering a worksheet with questions on differentiation:

> *'Also trouble with logarithmic implicit differentiation but actual problem seemed to be tidying up indices at the end and not the logarithmic or implicit part'.*

4.2.1.5 Logs

Data coded under *logs,* demonstrated a lack of understanding of the logarithm function and applying the rules of logarithms. The data, coded under *logs,* can be summarised and illustrated with the following examples:

Use of logs to solve equations

This area was shown as a difficulty where a student studying a Category B *Calculus* module was attempting a question on compound interest but appeared unable to complete the question even when the hint was given as seen in the *mathematical difficulty* below:

> *'[Student] wanted help with e^x = y. Tutor filled in eqn S = Pe^(rt/100); S= 90 P =30, t=7 and asked student to find r but student could not solve it. Tutor wrote log_a (e^x) = y and show it implied a^y = e^x and if base was e that log_e (e) =1 and tutor wrote log_e (30) + (.07r)= log_e (90).'*

Similar difficulty for a Level 3 module occurred but the student had recalled the method when reminded:

'Didn't know how to solve for t for an equation like 1233(1.5)^t = 1200*(2.5)^t. Told to apply ln to both sides and use log rules, student remembered that they had done that before.'*

Difficulty with answers provided for lecturers' worksheets

A student studying a Category B *applied mathematics* module displays an example of this when they had difficulty because their answer did not match the answer provided by the lecturer:

'Lecturer's answer was $\frac{log(2)}{k}$. *Student's answer was* $\frac{-log\left(\frac{1}{2}\right)}{k}$.'

4.2.1.6 Complex numbers

The code *complex numbers* included addition, subtraction, multiplication, and division of complex numbers:

Basic operations on complex numbers

How to add or multiply *complex numbers* and how to divide by a *complex number* were difficulties evident in *mathematical difficulties.* Here are two examples, the first from *a Level 1* Category C linear algebra module:

'How to get rid of the complex number from the bottom of a fraction by multiplying above and below by the complex conjugate, [one example covered] (1+i)/2(1-i).'

In this tutor entry from a Level 3 module, the tutor has indicated a problem with adding and multiplying *complex* *numbers* where a student had come to the MSC needing help with transformations of the complex plane:

> *'Transformations in the plane, adding complex numbers, multiplying them together.'*

These examples were from the same modules described above. First the Level 1 *Category* C module:

> *'Student didn't get the idea of the inverse of a non-zero complex number. The ideas of conjugate and modulus were covered and student was left to do a few examples.'*

This is a similar difficulty experienced by a student studying the Level 3 module:

> *'[Student] didn't understand that modulus of complex number was distance from origin on the argand diagram'.*

Separating complex expressions into real and imaginary parts

Once again this is evident in students studying either module, and this is a difficulty experienced by a student studying the Level 3 module:

> *'Decompose complex function into real and imaginary function. Didn't understand that a complex number 2+x+iy=(2+x)+iy and as such could not find conjugate to solve \frac{1}{2+x+iy} as a + ib. But student could find answer once given this information.'*

4.2.2.7 Summary of Algebra grouping and codes

Finally, the results explored in the algebra grouping are summarised in Table 4.3 below. Evidence of difficulties across modules and within modules is provided by choosing those codes from the *Algebra* grouping which were found to cause widespread difficulty for students. Their presence is demonstrated in modules which had the highest instance of difficulties in this grouping. Also, included are the number of students registered to each module.

It is clear from Table 4.3 below that students from certain modules, such as Category B and C *calculus* modules (see Table 4.1) exhibited smaller numbers but more widespread difficulties in the *Algebra* grouping. Whereas, students from modules in *linear algebra, discrete mathematics* and *number theory* displayed difficulties in a small number of specific areas.

It is also important to look at the class size. For example, looking at entries for The Level 1** (Category B) Calculus class, students enter this module with a minimum level of an O2 in the mathematics Leaving Certificate examination and would not have covered logs previously. Twenty-two visits relating to the code of *logs* seemed high however, when we look at the class size (Column 7) it is a relatively small percentage. Where the number of visits are high compared to the class size, the *mathematical difficulties* are evident in areas not previously covered by the students in secondary school. For example, this is seen in the 76 visits for the code of *discrete mathematics* from a class size of 114 students.

This will be explored further in answers to research question 3 and 4.

Table 4.3 Presence of codes(*) in the algebra grouping evidenced across modules

Modules	***Matrices***	***Discrete maths***	***Basic algebra***	***Indices***	***Logs***	***Number students registered***
Level 0 *Introductory Modules*	n/a	n/a	13	10	0	50
Level 1* *Calculus*	n/a	n/a	10	0	n/a	361
Level 1 *Calculus***	n/a	n/a	9	28	22	522
Level 1* *Calculus***	n/a	1	19	5	7	293
Level 1 Linear *Algebra***	61	n/a	1	0	0	305
Level 1* *Linear Algebra***	31	3	3	1	0	186
Level 1 *Number Theory***	n/a	27	2	5	0	61
Level 1* *Number Theory***	n/a	76	6	0	0	114
Level 1 *Applied Mathematics***	1	3	1	0	3	293
Level 2 *Calculus*	3	0	8	1	0	281

* Category A; ** Category B; *** Category C (see Table 4.1)
n/a not applicable

Matrices

Matrices was not applicable in six of the modules as seen in Table 4.3. *Mathematical difficulties* found with *matrices* were four or less for the 17 modules exhibiting difficulty in this area other than the two linear algebra modules, Level 1 Category B Linear Algebra and Level 1 Category C Linear Algebra. These modules show relatively high numbers of difficulty. Level 1 Category B Linear Algebra has a basic entry Leaving certificate at a lower level (O2) compared to an entry level of a H3 for Level 1 Category C Linear Algebra (see Table 4.1).

There were sixty-one visits in total, as seen in Table 4.3 above for the Category B linear algebra module with forty students attending. Two students visited on five occasions, two students on four, three students on three, but the majority of attendance was represented by single student visits. The main areas of difficulty for the Category B module were exhibited in row reduction and a Hot Topic in this area was organized for ten students registered to this module. Other areas of difficulty noted were, multiplication of matrices and finding determinants. A small number of difficulties related to the properties of matrices and determinants.

Students taking the Category C linear algebra module visited the MSC on thirty-one occasions. One student visited on three occasions but the majority of visits were single visits and twenty-three students in total, registered to this module, attended for help with *matrices*. The data showed students in the Category C module mainly sought help with understanding and applying the properties of matrices. Other areas of difficulty evident but less frequently were, finding parametric solutions and finding determinants.

Discrete mathematics

Students from thirteen individual modules visited the MSC over the eight weeks of data collection period with difficulties coded as *discrete mathematics*. The majority of modules were represented by four or less students visits. There were nine visits each from students registered to two second level modules the first module was graph theory and the second an algebraic structures module. Five students attended with difficulties exhibited for the former module with three or less visits by a single student. Data for the latter module showed one student had five individual visits with other students visiting at most on two occasions. However, the data coded as *discrete mathematics* were mainly evident with students taking two Level One number theory modules as seen in Table 4.3 above.

The Category B module, with a total of twenty-seven visits, was represented by visits from fourteen individual students where a single student visited six times for a number of different areas, three students visited three times but the majority of visits were single visits. It is noticeable that these modules had relatively small number of students registered to the module, as seen in Table 4.3 but their student attendance numbers were high. The dominant area of *mathematical difficulty* exhibited by students in this module was proof by induction. Other areas were binomial expansion, use of the inclusion/exclusion principle, modular arithmetic proofs, and the Euclidean algorithm.

The Category C module required a higher level of mathematics on entry and had the highest number of visits, at seventy-six, recorded. Forty-one individual students visited the MSC over the eight-week period for help with this module with three individual students visiting six, five and four times respectively, each visiting for a number of different areas of number theory, but the majority of students visited just once for help with this module. This module, also, had a relatively small number of students registered to the class, as seen in Table 4.3 but their student attendance numbers were high. The data recorded for this module showed that the highest number of *mathematical difficulties*, exhibited by the students in this module, were in the area of proof by induction. Three other topics displaying high attendance were proofs relating to the binomial theorem and binomial expansion, modular arithmetic, and applications of inclusion/exclusion principle. Euclidean algorithm proof by counter example (5%) with smaller percentages in other areas of *discrete mathematics* such as sequences, series and Fermat's little theorem.

Basic algebra

The most common *mathematical difficulty* across the above modules was shown to be in *basic algebra* evident in all of the above modules. In most cases, the numbers are small when seen in relation to the overall class sizes. For the Level 1 Category C Calculus module above, fourteen students required help, with one student looking for help in this area on three separate visits with three students coming twice. The main areas of difficulty for this module were simplifying algebraic expressions and adjusting (sinax/x) to (asinax/ax) when finding trigonometry limits. A level zero introductory calculus module had thirteen visits which although, the number in the class is small, it is expected that students at this level would have difficulty with basic algebra. *Mathematical difficulties* exhibited for the Level 2 calculus module were mainly in three areas: simplifying more complex algebraic expressions or changing an expression to a different form, such as, $(1/(2 + e^{(-x)})) = e^x/(2e^x + 1)$; algebraic division related to solutions of cubic equations; also, missing solutions for example $y=0$ in following example $y^3 = 2y$.

Indices and logarithm

The codes of *indices* and *logs* are often inter-related and for this reason they are described under one heading. The highest number of visits for the codes of *indices* and *logs* were evident in visits from a Level 1 Category B calculus module as seen in Table 4.3 above with twenty-eight and twenty-two visits respectively. These numbers included eleven students attending a *Hot Topic* covering exponents and logs organized for this module. Main areas of difficulty were fractional and negative indices and understanding the principle and properties of logarithms. It was not possible to conclude that students in, for example, the linear algebra or *discrete mathematics* modules would not have had similar difficulties if studying different modules. Perhaps, they were not evident in these situations because the nature of the module did not require knowledge of these mathematical areas. The introductory calculus module was different from other modules as the lecturer carried out extra tutorials for these students in the MSC and entered on the database specific difficulties exhibited by the students. This module covers both indices and logarithms but difficulties only with *indices* were indicated. The number of algebra difficulties, at ten, was relatively high with respect to the number of students registered to the module. The main problem areas of difficulty were in very basic algebra such as expanding 2(x + 3)(x + 4) as (2x + 6)(2x + 8).

Complex numbers are not included in the Table 4.3 as *mathematical difficulties* for this code are not widespread across modules. They are mainly evident in Level 1 Category C linear algebra and Level 3 modules.

4.2.2 Grouping 2 – *Calculus*

Eight of the codes from Table 3.8 are classified under the grouping of *Calculus*. Table 4.4 below displays the number of *mathematical difficulties* for each code.

Table 4.4 Codes and number of mathematical difficulties in the calculus grouping

***Calculus* grouping**	**Number of *Mathematical Difficulties***
Differentiation	71
Integration	64
Graphs	63
Partial differentiation	58
Functions	48
Limits and continuity	47
Critical points	22
Domain and range	11
Total *mathematical difficulties*	**384**

This grouping represents over 21% of the total *mathematical difficulties*. A description of each code with more than 30 *mathematical difficulties* is described below, while a brief description of remaining codes is given in an appendix. (See

Appendix C for further details of codes numbering less than 30). The Calculus grouping represented approximately 21% of all the *mathematical difficulties* recorded over the eight-week research period.

4.2.2.1 Differentiation

The *mathematical difficulties* coded under *differentiation* are summarised and illustrated in the following examples:

Use of product, quotient or chain rules

First and second derivatives of linear, quadratic and cubic functions are covered by rule in the Ordinary Level Leaving Certificate Mathematics syllabus. Product, quotient and chain rules are not covered in the Ordinary Level but are covered in the Higher Level. This is an example from a student in a Category B module where the minimum entry level for students taking this module is the Ordinary Level (See Table 4.1):

> *'Differentiation, student was unsure how to recognise when to use quotient or product rule and sometimes didn't see that you might have to apply chain rule inside product rule.'*

This example demonstrates the difficulty of a student in a Category C *Calculus* module where the entry requirement for this module is a H3 in the Higher Level Leaving Certificate mathematics paper (See Table 4.1):

> *'The student required a bit of extra confidence using the chain rule and quotient rule. Examples covered: y=(1+8x^3)^4; Y=3(cube root(x + 1); f(x) = e^(2x)/(1 + ln(3x))*

Differentiation of logs or exponentials

The Ordinary Level Leaving Certificate Mathematics syllabus unlike that of the Higher Level does not cover differentiation of logs and exponentials. However, the recommendation for this Category C *calculus* module is that students have obtained a minimum H3 in Higher Level Mathematics in the Leaving Certificate. The following is a *mathematical difficulty* relating to a student taking this module:

> *'Trouble with differentiating exponentials, how to use chain rule for exp(0.01x), Confused when had to* use product rule and chain rule in same problem. f(l) = 700l e^(-0.02l). Trouble with *differentiating logs, was able to get question out when rule for differentiating log was explained.'*

Implicit differentiation

Almost half of the *mathematical difficulties* for students studying a specific Category C *calculus* module demonstrated how they had difficulty in this area. The following is an example of one entry:

> *'How to find dy/dx by implicit differentiation of x^2 + y^2 =cosh^-1y. Student was confused because y didn't appear on its own on left hand side so didn't know where to start. Actual differentiation (other than implicit) was fine.'*

4.2.2.2 Integration

Students' understanding and execution of various integration techniques is coded as *integration.* These techniques include integration by substitution and by parts. Integration is not covered in the Ordinary Level Leaving Certificate, the Higher Level covers – integration of sums, differences and constant multiples of functions of the form: x^a where a $\in$ Q; a^x where a $\in$ R, a>0; sin(ax) where a$\in$R; cos(ax) where a$\in$R and to determine areas of plane regions bounded by polynomial and exponential curves. The *mathematical difficulties* arising in the code *integration* can be described as follows:

Integration by substitution

This was the most common area of difficulty found in the code of *integration.* Forty-three percent of the *mathematical difficulties* were found here. These were evident in both algebraic and trigonometric substitutions and were mainly seen in *mathematical difficulties* for students studying a Category C calculus module. Here is an example:

> *'Student sought help with integration by substitution question, difficulty in spotting how to relate the given choice of substitution to the integrand (which was not trivial). Having been guided to the relation, the student could finish the question using their knowledge of basic integrals, also discussed trig integrals.'*

Integration by parts

Thirty-one percent of the *mathematical difficulties* for *integration* related to a request for help with integration by parts. In a number of these, students made simple errors that led to difficult calculations. Another problem area was that students did not see that their original integral, after a number of repetitions, also appeared on the right hand side of the equation. This is an example:

> *'How to do integration by parts of exp(-2x)sinhx dx. Went through how to do int by parts twice to get exp(-2x)sinhx dx again and solve for it.'*

4.2.2.3 Graphs

Coded as *graphs,* are students' issues in relation to the sketching of functions and the identification of functions or reading regions of increase or decrease, including critical points, from given graphs. The main difficulties represented in this code could be classified as follows:

Sketching graphs

In a number of tutor entries it was evident that students had difficulty plotting graphs. For example plotting graphs of linear functions:

> *'How to graphically show that two functions reach equilibrium at a certain point, problem was plotting a line, went through a quick example and student was fine: P = 47b- 5q_d and P = 2q_s + 19.'*

Sketching quadratic functions was a particular problem for students studying both a *calculus* and an *applied mathematics* module both at the level of Category B (See Table 4.1). Students studying the *calculus* module also had difficulty with areas of increase and decrease and finding tangents. A *Hot Topic*, at which eleven students attended, was organised for a Category B *applied mathematics* module. The *tutor entry* for the *Hot Topic* was as follows:

> *'Hot Topic on graphing and visualising functions, student had difficulty sketching functions, particularly ones with asymptotes. (Tutor) went through method of finding roots and critical turning points to help graph functions and how to recognise and find asymptotes.'*

The problems, for students of Level 3 and 4 modules were difficulties with functions and graphs in their area of study. It is noticeable that students at these levels sometimes simply needed a reminder of the method. A difficulty for three Level 4 students, is seen below:

> *'There were some small issues with how to rearrange e.g. Qa=P*Qb and plot Qa vs P but students were just a little rusty and remembered once tutor went over it quickly.'*

Reading values and identifying functions from given graphs

This difficulty was mainly evident where students studying a Category B module had problems reading the value of critical points or areas of increase and decrease from graphs displaying the derivative of quadratic functions. Identifying various functions when given a number of graphs was also a problem. Here is an example of a student's query on a graph of f'(x):

> *'Given a graph of f '(x), how to read off the areas of increase, decrease and max and min points of f(x). Student was having trouble as they were mistaking it for the graph of f(x). Went through how to (roughly) convert the f '(x) to the f(x) graph and so read off the inc/dec/max/min and then how to read them straight off the f'(x) graph once they understood better.'*

4.2.2.4 Functions

Included in the code *functions* were student difficulties that recorded issues with understanding notation and the meaning of function and also, working with composite functions. The following is an example of a query from a student studying a Category A module (See Table 4.1):

> *'Student had problems solving basic questions in quadratic equations. They could find roots but were unable to realise how to rewrite questions into a root finding problem. Find g(x) = (x^2 -4x +5) such that (x,-9)is on the graph. tutor told student that this meant x^2 -4x +5 =-9. Tutor said student also had a problem .. f(x) = 6/g(x) and g(x) = x^2 -3x -4 although student knew f(x) was not defined when g(x) =0 but could not apply this to the question'.*

and a student studying a C*ategory* B module, a large mathematics module designed for non-mathematics majors had the following difficulty:

> *'Problem about finding maxima and minima, the student got stuck when they obtained f''(x) = 2. They said they couldn't substitute x in since there was no x on the RHS. Once it was explained that f'' was 2 for all values of x in f(x), the student was fine.'*

4.2.2.5 Partial differentiation

The main difficulty experienced by students was to understand that when differentiating with respect to a specific variable, the remaining variables must be treated as constants. Students also had problems in relation to the use of the product, quotient and chain rules when differentiating functions of several variables. *Partial differentiation* is not on any Level 1 module syllabus in UCD This is a *mathematical difficulty* experienced by a student in a Level 2 module:

> *'Student was answering a question on partial differentiation. There were 2 problems. The first was due to an inability to see which elements were constant in each case . . . Letting the original 2 var. fcn = w and then first deriv. dw/dx seemed to*

cause them problems. So showed student another way to represent dw/dx as fx (x,y) etc. they seemed to find this easier.'

4.2.2.6 Limits and continuity

Mathematical difficulties coded as *limits and continuity* include descriptions of methods used in finding limits and showing continuity in both one- and two-variable functions. The following is an example taken from a Level 2 *calculus* module:

'How to find the limit of $\frac{y^3}{x^2+y^2}$ $as\ (x,y)\ \rightarrow (0,0)$. *Trouble with why is* $\frac{y^2}{(x^2+y^2)} \leq 1$ and therefore why $\frac{y^2}{(x^2+y^2)}y \leq 1$.'

4.2.2.7 Summary of the calculus grouping and codes.

Table 4.5 below gives a summary of the codes with more persistent *mathematical difficulties* across modules. The modules are chosen to demonstrate those modules which had higher numbers of these difficulties.

Table 4.5 Presence of codes in the calculus grouping within and across modules.

Module	Limits and continuity	Differen,tio	Integr'n	Graphs	Funct'ns	Pardiff	Number of students registered
Level 0 Introductory Module	n/a	n/a	n/a	1	6	n/a	50
*Level 1** Calculus	4	4	n/a	2	3	n/a	361
*Level 1*** Calculus	1	27	n/a	21	13	n/a	522
*Level 1**** Calculus	7	24	28	2	10	n/a	293
*Level 1*** Applied Maths	12	1	1	14	1	n/a	65
Level 2 Calculus A	0	2	2	5	2	36	281
Level 2 Calculus B	9	3	1	1	1	11	133
Level 3 Applied mathematics	0	0	16	0	2	0	65

* Category A; ** Category B; *** Category C (see Table 4.1)
n/a not applicable

Limits and continuity

Limits and continuity were not covered in the syllabus for the introductory module. The number of visits for *mathematical difficulties* related to this code were relatively few. Students from ten modules attending the MSC over the eight-week period exhibited difficulties in this area with a total of forty-seven visits. The highest number of difficulties were seen in a level one applied mathematics module with twelve visits. Eleven of these students attended a Hot Topic specifically for issues in this area. A single student also attended with the following problem find the limit of e^-n as n tends to infinity.

There were no subsequent visits for *mathematical difficulties* in this area by the students studying this applied mathematics module.

Students taking a Level 2 module presented with nine *mathematical difficulties* categorized under this code with one student visited three times and other students had single visits. The principle areas of difficulty were related to finding limits for multi-variable functions, in particular understanding the use of limits along various paths.

Two Level 1 calculus modules were the next highest for *mathematical difficulties*. The Category C calculus module with seven visits displayed difficulty with trigonometric limits and those requiring division of rational expressions by the highest power. All seven visits were by individual students. The Category A calculus module, with four visits exhibited difficulty with finding limits by cancellation, for example finding the limit of (x^2 – 1)/(x – 1) as x tends to 1. These were also individual student visits.

Differentiation

The total attendance in relation to help for this code was seventy-one visits with students from a total of thirteen modules. Attendance was high for two modules, a Level 1 Category B Calculus module and a Level 1 Category C Calculus module.

The highest number of individual student visits for the Category B module was four by an individual student and two students visited twice but the majority of visits were single visits. The major areas of difficulty displayed for the Category

B module, with twenty-seven visits were: converting the first differential graphs of functions to the original function, use of product, quotient and chain rules, differentiation of logs and exponentials and the concept of tangents.

The number of visits for students in the Category C module was twenty-four. One student attended three times, four students twice and the balance were single visits. The main areas of difficulty exhibited by students attending the MSC for assistance with this module were: implicit differentiation, the use of product, quotient and chain rule, and differentiation requiring the application of logs for example differentiation of F(x) = x^x.

Integration

There were sixty-four visits for this code from thirteen individual modules. Similarly, to the *differentiation* code, *integration* difficulties were seen as high for two modules. Integration was not covered on the syllabus for the Category B module discussed in *differentiation* above.

The module with the highest number of visits, at twenty-eight visits, was the same Category C module discussed previously in *differentiation*. One student visited the MSC five times for assistance with this module, three students had two visits and the remainder were visits by individual students. The main areas that students required assistance for this code, were in order of most frequent to less frequent: integration by parts, integration by substitution, trigonometric integration and basic monomials.

The other module with a high number of visits in relation to a difficulty with i*ntegration* was a Level 3 applied mathematics module. There were sixteen student visits for this code with two students visiting twice and the remainder were single visits by individual students. Areas of difficulty displayed were: trigonometric integrals such as finding the integral of $\sin^{a} x \cos^{b} x$ by substitution, integration by parts, and basic integration such as the integral of $\cos^{2} x$.

Graphs

There were a total of sixty-three visits for this code. Nineteen individual modules sought assistance. Two modules had a high number of visits. These were a Category B applied mathematics module and the Category B calculus module as previously seen in differentiation.

There were twenty-one student visits for the calculus module with fourteen students attending. Three students visited on three occasions, one student visited twice and the remaining students only visited once. The main areas of difficulty were graphs of logs and exponential functions identifying quadratic functions from a given graph and drawing tangents.

Eleven students attended the MSC for assistance with the applied mathematics module with fourteen visits in total. There was a *Hot Topic* organised for eleven students covering sketching quadratic functions in particular addressing finding asymptotes.

Functions

The total number of visits for the code of *functions* were forty-eight comprising of visits from thirty-nine individual students. Students from thirteen modules exhibited difficulty in this code. There were two modules with a relatively high level of student attendance. These were a Category B and Category C Level 1 calculus modules as seen previously for the code of *differentiation*.

The Category B module was represented with thirteen visits from nine students. Two students visited on three occasions and remainder of the students attending for this code had single visits. The areas of difficulty exhibited were quite varied but included: composition of functions, the Mean Value theorem, and inverse hyperbolic functions

Of the seven students taking the Category C module, one student visited the MSC for this code three times, another student visited twice and remainder of student visited only once. Finding areas of increase and decrease in given functions, understanding exponential functions and identifying the sign of 'a' in quadratic functions of the form f(x) = ax^2 + bx + c from a given graph.

Partial differentiation

There were fifty-eight visits in relation to this code. Thirty-two students from five modules sought assistance for *mathematical difficulties* with *partial differentiation.* There were three or less visits for three of these modules.

Seventeen students from a single module, a Level 2 calculus module for engineering students, visited a sum-total of thirty-six times with one student attending on eight occasions, another on five and three students attended three times. The main areas of difficulty were: failing to treat other variables as constants, f(x,y,z)=2xyz example stating differential of this function was 2, use of product and chain rule in differentiation of multivariable functions, finding critical points and drawing level curves.

There were eleven visits to the MSC for help with *partial differentiation* from another calculus module. Two students attended on two occasions and the balance were visits by individual students. The main areas of difficulty were finding critical points and directional derivatives.

4.2.3 Grouping 3 - Applied Mathematics

Table 4.6 Codes and number of mathematical difficulties in the applied mathematics grouping

***Applied Mathematics* grouping**	**Number of *Mathematical Difficulties***
Vectors	142
Mechanics	108
Trigonometry	45
Total *mathematical difficulties*	**295**

The three codes *vectors*, *mechanics* and *trigonometry* were placed together under the *applied mathematics* grouping. Table 4.6 above gives the number of *mathematical difficulties* experienced by students in *these areas.* The *applied mathematics* grouping represents approximately 16% of all the *mathematical difficulties* recorded over the eight-week research period.

4.2.3.1 Vectors

The difficulties coded in *vectors* were mainly evident in the section basic understanding of *vectors,* as described below. Difficulties relating to the resolution of *vectors*, orthogonal projection, dot and cross products of *vectors* were also evident.

Basic understanding of vectors

This topic included understanding the difference between vector and scalar quantities, triangle and parallelogram rules, finding perpendicular vectors, and equality and magnitude of vectors. Approximately 50% of *mathematical difficulties* for the code of *vectors* could be described as representative of this area. This is an example of help sought by a student studying a Category C *applied mathematics* module (see Table 4.1):

> *'Student had no understanding of vectors, did not know that distance or time were not vectors. Question gave a plane driving East from A for 224 Km (in 23 mins) and then north 482 km (24 mins). Showed student how to convert units to velocity and solve using triangular rule for vectors. Student realised they could use Pythag. but found difficulty getting the resultant angle.'*

Calculating resultant vectors

This was also a major problem for students coming to the MSC for assistance with the code *vectors* and this was evident in a high number of *mathematical difficulties*. The following is an example of a problem experienced by two students in a Category B *applied mathematics* module prior to the organisation of a *Hot Topic* for the module:

> *'Went through problem of resolving forces of car being pushed up a hill, F= 10N 30 deg to the horizontal, didn't know to use hill as flat x axis and y as perpendicular y axis. Had trouble resolving forces into x and y components, knowing whether to use cos or sin so went over that. Used 90 deg triangle, sin, cos tan. Also got confused when there were many things to resolve at once, trouble finding resultant force (summing x and then y components).'*

Orthogonal projection, dot and cross product

The main difficulties for students in this area were understanding the methods used to calculate dot and cross products and understanding the concept of orthogonal projection. This is an example where four students studying a Category C *applied mathematics* module experienced difficulty:

> *'Working on cross product problems, unsure of how to calculate, question was actually about 3 vectors that were co planer and finding the missing component of one of them. It was easier to not use cross product in the end. (2,1,-2) + b(-3,1,-2) = (1, u_y,4) find u_y . Tutor also showed student how to calculate the cross product of (2,1,-2) and (-3,1,-2).'*

4.2.3.2 Mechanics

Mathematical difficulties in the code of *mechanics* represented topics including equations of motion; moments and force; differential equations; simple harmonic motion; standing waves; and moments of inertia. The following are examples of the higher frequency topics:

Equations of motion

Issues in this area were mainly the use of Newton's laws of motion:

> *'Projectile thrown upwards from a cliff with a certain velocity. Some conceptual problems imagining the situation. Also, what is the velocity at h=0, using s(t) to find time at 0 then using v(t) to find velocity at 0. Didn't really think of using multiple equations to get the information.'*

Differential equations

> *'The students wanted help with transforming a nonlinear differential equation into a linear one. They already had solutions, but could not understand them fully. Once the equation had been transformed into a linear one, they were OK from there.'*

Simple harmonic motion and standing waves

> *'Simple Harmonic Motion, student confused between the natural length of an elastic string and the extension of the string x.'*

4.2.3.3 Trigonometry

Students' difficulties with the code *trigonometry* included very basic trigonometry as described below and applications of trigonometric identities where they arose in other mathematical areas.

Basic trigonometry

The difficulties, experienced by students in this area, were applying the ratio of sides to cosine and sine in Pythagorean triangles, cosine and sine as co-ordinates on the unit circle and the meaning of the inverse of trigonometric functions. Over 40% of *mathematical difficulties* involved difficulties in these areas. Below is a typical entry for the code of *trigonometry* for students studying a Category A *calculus* module:

> *'Student came in asking about trig problems finding an angle given two sides or finding a side given an angle and a side. The student was fine after the method was explained.'*

Trigonometric identities

This area covered the use of trigonometric identities in providing solutions to various mathematical problems. Difficulties included an inability to apply known identities to simplify calculations or find given solutions to worksheet problems. A number of these difficulties were observed in higher level modules, particularly, one Level 3 module. The majority of difficulties were experienced by students in Level 1 modules. In the following example a student in a Category C *linear algebra* module sought help as follows:

> *'Given complex number a + bi at an angle theta to the horizontal [tutor] showed Sin(theta) = b/(a^2 + b^2)^0.5 and cos(theta) = a/(a^2 + b^2)^0.5. Tutor said main problem [student had] was knowing 1/2 angle tan formula tan(A/2) = sin(A)/ (Cos(A) + 1).'*

Table 4.7 Presence of mathematical difficulties in the applied mathematics grouping, within and across modules

Modules	***Trigonometry***	***Mechanics***	***Vectors***	***Number of registered students***
Level 1* Applied Maths	0	18	2	65
Level 1** Applied Maths	8	7	19	97
Level 1 Applied Maths***	13	18	46	293
Level 1 Physics* module**	3	20	12	284
Level 1 Linear Algebra A**	0	1	18	305

* Category A; ** Category B; *** Category C (see Table 4.1)
n/a not applicable

Presented, in Table 4.7 above, is evidence of students' difficulties as exhibited for the three codes, in modules exhibiting high numbers of student attendance.

4.2.3.4 Vectors

The data showed there were one hundred and forty-two visits for the code of *vectors* over the eight-week data collection. This represented visits from eighteen separate modules and one-hundred–and-nine students seeking assistance. One student visited the MSC five times, two students four, one student three but the majority of students visited on one occasion only for help relating to the code of *vectors*.

High levels of student attendance were seen from four Level 1 modules, students in a further four Level 2 modules had attended less frequently and students from the other ten modules visiting for assistance with *vectors* exhibited very few visits.

The module with the highest number of students attending for help with *vectors* was a Category C applied mathematics module with forty-six visits to the MSC by thirty-six students. One student sought help on four occasions, one on three, five on two but majority were once-off visits. The main areas of difficulty exhibited by the students were as follows: Resolving two and 3-dimensional vectors into their horizontal and vertical components. Finding an unknown component given the necessary information, finding cross product and applying these results to answer worksheet questions.

A Category C applied mathematics module was represented by nineteen visits from eleven individual students, two of these students visited the MSC three times over the eight-week period, four students twice and five were single visits by individual students. A *Hot Topic* was organized for this module at which only six students attended. *Vector* topics covered in this *Hot Topic* were resolving vectors to find resulting components and the dot product. The two students that had visited on three occasions for assistance with this module were attendees at the Hot Topic. Other areas for which students sought help included: magnitude of a vector, finding perpendicular to a vector, and calculating the unit vector. The relatively small size of this class is worth noting.

Eighteen visits for assistance with this code were made by students registered to a Category B linear algebra module. Three of these students visited twice and twelve students attended on a single occasion. The areas of difficulty exhibited by these fifteen students were varied in nature but the main difficulties were with: finding the length of a vector, the vector equation of a line, the dot product, the perpendicular to a given vector and orthogonal projection onto a line.

Ten students attending a Category C physics module sought help for the code *vectors*. Two of these student attended twice and eight visited on one occasion only for help with *vectors.* The main areas of difficulty were understanding the meaning of a vector and resolving vectors into parallel and perpendicular components.

4.2.3.5 Mechanics

The data revealed that there were one hundred and eight visits for the code of *mechanics* over the eight-week data collection. This represented visits from eighty-five individual students registered to one of eighteen discrete modules.

High levels of student attendance were seen from four Level 1 modules, students in a further Level 1 and one Level 2 modules had attended less frequently. Students from the other twelve modules, visiting for assistance with the code of mechanics, exhibited very few visits.

The module with the highest number of students attending for help with mechanics, on twenty separate occasions, was the same Category C physics module, from which ten students visited the MSC for difficulties in relation to *vectors.* Four of these students sought help on two occasions and twelve students had single visits for the code of mechanics. The main areas of difficulty exhibited by the students were answering worksheet problems: on Newton's laws of motion, two-dimensional collisions, conservation of energy and momentum and standing wave problems.

The Category C applied mathematics, from which thirty-six students attended the MSC for help with *vectors*, was represented by eighteen visits from sixteen individual students, one of these students visited the MSC three times over the eight-week period, the balance were single visits by individual students. Areas for which these students sought assistance were: Moments of inertia, calculating moments and equilibria, seeking help for project on the compound pendulum and finding the centroid of an irregularly shaped object.

Eighteen visits for assistance with the code of mechanics were made by students registered to a Category A applied mathematics module. One of these students visited twice and sixteen other students attended on a single occasion. The areas of difficulty exhibited by these seventeen students were varied in nature but the main difficulties were approaching answers to worksheet problems in the following areas: the stability of fixed points, initial value problems, setting up answers to questions requiring the use of the integrating factor method and understanding the meaning of one dimensional fields.

Ten students (not shown in Table 4.7) attending a Category B physics module sought help for the code of *mechanics* but not for assistance with *vectors* or *trigonometry*. Two of these student attended twice and eight visited on one occasion only. The main areas of difficulty were answering worksheet questions on Newton’s equations of motion, simple harmonic motion and fluid pressure.

4.2.3.6 Trigonometry

Thirty-seven students representing ten modules made a total of forty-five visits to the MSC for assistance with the code of *trigonometry*. One student visited the MSC on four occasions another on three, three students on two and thirty-two students had just a single visit for assistance with *trigonometry.*

The highest attendance of students, at thirteen visits, from a single module was for the Category C applied mathematics module from which students also attended for help for both the codes of *vectors* and mechanics as described above. The main area of difficulty for these students was understanding trigonometry necessary to resolve forces. Eight students from the Category B applied mathematics module six of whom had attended the Hot Topic organised for this module and described under the code of *vectors* exhibited similar difficulties in the use of trigonometry when resolving components. Six students attending a Category C linear algebra module (not shown in Table 4.7) visited the MSC for difficulties with *trigonometry* but not for *mechanics* or *vectors*. One student visited on two occasions. The main difficulties for students studying this module were: finding the argument and the exponential form of a complex number, and the use of De Moivre's theorem.

4.2.4 Grouping 4 - Statistics

Two codes were included in the grouping of *statistics*. Calculations in the area of statistics vary depending on whether the data are discrete or continuous. Where knowledge of content or calculations use discrete data, for example in calculations for Binomial or Poisson distributions, difficulties have been classified as *discrete distributions*. If the difficulties relate to continuous data, such as applying calculations of Normal or t-distributions, these were classified as *continuous distributions*. All *mathematical difficulties* included under the grouping of *statistics* are coded in either *discrete distributions* or *continuous distributions.*

Table 4.8 gives the codes in the Statistics grouping and includes the number of *mathematical difficulties* exhibited by students for these codes.

The Statistics grouping represents approximately 8% of all the mathematical difficulties recorded over the eight-week research period.

Table 4.8 Codes and number of mathematical difficulties in the statistics grouping

***Statistics* grouping**	**Number of *mathematical difficulties***
Continuous distributions	108
Discrete distributions	36
Total *mathematical difficulties*	**144**

4.2.4.1 Continuous distributions

Some of the principal difficulties experienced by students in the code of *continuous distributions* were in the following areas: calculating mean; median; standard deviation; standard error; z-scores and t-scores; confidence intervals; hypothesis testing; understanding the use of normal or t-distributions and reading tables.

Students studying modules with a research component, at Level 3 or 4, required assistance, in some cases, with basic knowledge needed for the study of statistics. This was evident in 14 *mathematical difficulties.*

The majority of *mathematical difficulties* were experienced by students in three modules, one a Level 1 module and the others were Level 2 modules but covering similar statistics. The students taking the Level 2 modules were studying statistics for the first time. The Level 2 modules are distinguished here as Module 2A and Module 2B.

Understanding Normal and t-distributions and reading tables

Under this heading are included understanding the use of z- and t-scores and finding probabilities by reading values from the respective tables. This was the area accounting for approximately 33% of the *mathematical difficulties* for the code of *Continuous distributions* and was experienced by students in the Level 1 module and the two Level 2 modules considered above. There was also a *Hot Topic* covering the same topic organised for the Level 1 module. This is a typical example of the difficulty the students experienced:

> *'Student was having trouble with using the normal distribution tables to calculate probabilities. They had correctly converted x = 40 into a z score using the given mean and st. dev. but they were unsure of how to proceed using the tables. I showed them a diagram and explained how the symmetry of the normal curve helps us work out the correct probability. Tutor drew diagram to show P(z< -1.7625) = P(z > 1.7625) = 1- P(z < 1.7625).'*

Hypothesis testing

Twenty-five percent of the *mathematical difficulties* were for help sought by students in relation to hypothesis testing. The majority of these were for students studying one of the Level 2 modules described above. Here is an example of one *tutor entry*:

> *'Student did not understand matched and unmatched samples . . . Student had a problem with hypothesis testing and a problem with finding the correct formula to use . . . Student did not understand one tailed or 2 tailed hyp testing, test_(crit) or test_(statistic) and how you use them. Student did not understand acceptance and rejection regions or how to use t tables.'*

Confidence intervals

Nineteen percent of the *mathematical difficulties,* experienced by students, were in the area of confidence intervals. Students studying the same Level 2 modules were the major visitors for this area of statistics. This is an example of assistance sought by four students taking this module:

> *'Differences between confidence intervals for paired versus unpaired samples, tutor*
> *discussed notation for confidence intervals.'*

4.2.4.2 Discrete distributions

The code *discrete distributions* can be described under two main headings as follows:

Basic Concepts

Classified under this heading are the *mathematical difficulties* where students needed assistance in understanding simple probability, the basic laws of probability or Bayes Theorem. This is a typical *tutor entry* for the code of *discrete distributions* for students studying a Category B Level 1 *statistics* module:

> *'What is a discrete variable? If 200 out of a population of 1000 is female, how many females should be surveyed out of 50 people. What makes a good survey? What does n! mean, what does* $i = 1 SUM N$ *mean . . . If a survey has a 10% response rate, how many do you need to hand out to get 100 respondents.'*

Distributions

Difficulties with distributions such as Binomial, Hypergeometric and Poisson are included under this heading. These distributions were the major areas of difficulty for the code of *discrete distributions* and were represented in 80% of the *mathematical difficulties* for this code. The following is an example of this difficulty as experienced by a student in the Level 1 module described above:

> *'If the average number of radioactive particles detected per millisecond is 3, what is the probability that at most two will be detected in a given millisecond. Student recognised that it was a Poisson distribution. Student was given spreadsheet with all probabilities worked out so went through what each box calculated and which one to pick. Went through that if you find P(<=X) then P(>X)=1-P(<=X).'*

Table 4.9 below summarises the main areas of difficulty within the codes *discrete distributions* and *continuous distributions.* These problems were mainly experienced by students in the three modules discussed above. The *mathematical difficulties* of the *Statistics* grouping are summarised by separating these difficulties into five main areas and showing their presence in these modules.

The statistics syllabi for both Level 2 modules seen in Table 4.9 are taught at the same basic standard of statistics as the Level 1 module shown and cover similar areas of statistics. Students taking the Level 2 modules will not have taken any Level 1 statistics course previously. For this reason, these three modules are evaluated as equivalent in the level of statistics covered and are treated as if they were all Level 1 modules in the analysis of the data. None of these three modules had the Higher Level Leaving Certificate mathematics as a requirement to sit these modules.

Table 4.9 Presence of mathematical difficulties in the statistics grouping within and across modules

	Basic probability Discrete distribut'n	*Mean Standard Deviation CLT*	*Normal & t-distribut'n*	*Hypothesis Testing Confidence Intervals*	*Reading Statistical Tables*	*Number of Registered students*
*Level 1** module*	17	22	26	3	23	519
Level 2A Module	2	6	10	10	8	295
Level 2B Module	10	8	11	24	11	221

Other Level 1 and Level 2 statistic modules studied at the university are covered at a higher level of statistics. However, these are not included in the Table 4.9 as the numbers of students attending from these courses are very small.

4.2.4.3 Discrete distributions

Thirty students, registered to ten modules, visited the MSC on thirty-six occasions for assistance with *discrete distributions*.

The highest attendance, in this area, was for students taking the Level 1 module. A Hot Topic, covering *discrete distributions*, at which nine students attended, was organized for these students early in the semester. There had been no visit from the students registered to this module prior to the Hot Topic. The Hot Topic covered Binomial, Hypergeometric and Poisson distributions and also how to read the statistical tables in relation to these areas. The students attending did not have difficulty with basic probability. In the seven weeks following the Hot Topic, there were eight visits to the MSC for assistance with *discrete distributions*, by students registered to this module. These eight visits comprised two students attending on two occasions and four made single visits. None of these six students had attended the Hot Topic session.

Seven students registered to the Level 2B module attended the MSC for assistance with *discrete distributions*. Three of these student visited on two occasions the other students made single visits. The main areas of difficulty for these students were basic probability, such as for tossing coins, and also binomial distribution. One student asked for help using the binomial statistic tables.

Only two students attending the Level 2A module sought help for *discrete distributions*. Binomial distribution was the area covered for these students.

4.2.4.4 Continuous distributions

Eighty students, registered to fourteen different modules, visited the MSC a total of one hundred and eight occasions, for assistance with *continuous distributions*, over the eight-week period of the data collection. Attendance by students from the same three modules, as described in *discrete distributions*, similarly showed the highest visits to the MSC for *continuous distributions*. Attendance by students from other modules were relatively few.

Twenty-six students from the Level 1 module attended for assistance with this code. The data showed that, over the eight-week period, there were thirty visits by these students with nineteen of them attending a Hot Topic organized for the module. This was a higher number of students than normally recommended for attendance at a Hot Topic. Three of the five students that had not attended the previous Hot Topic but had visited the MSC shortly afterwards, attended the second Hot Topic. There were eight visits by students from this module to the MSC after the Hot Topic, with three of these visits relating to students who had not attended the Hot Topic. The principle topics covered over the eight-week period were centrality of data, skewed data, standard deviation and variance, normal and t distributions and reading statistical tables. These were covered during the Hot Topic. Assistance with hypothesis testing and confidence intervals was less frequently observed. The possible reason for this is they would be the last area covered for the syllabus and students may have come in the last week of the semester for help with these topics or decided not to cover them.

Thirty-two visits from twenty-two students registered with the Level 2B module attended the MSC over the eight-week period. One student visited on three occasions, eight students on two and the remaining thirteen students attended on a single occasion. The major area of difficulty with which students were assisted on twenty-four occasions was for hypothesis testing and confidence intervals. Other topics, which showed significant difficulty for these students, were normal and t-distributions including reading statistical tables. Eight visits related to *mathematical difficulty* with mean, standard deviation and the central limit theorem.

When we consider the attendance of students from these three modules the number of visits and the number of students attending for the Level 2A module were less with twenty-five visits from thirteen students. Also the frequency of visits by each student was higher. Two students attended on four occasions, three on two occasions, two on two occasions and seven made visits on one occasion only. The *mathematical difficulties* for which these students attended the MSC were similar to those for the other modules described above - normal and t-distributions including reading statistical tables, hypothesis testing and confidence intervals and slightly lesser number of visits for mean and standard deviation.

4.2.5 Grouping 5 - *Advanced*

The *advanced* code represents 252 *mathematical difficulties,* 14% of the total number of *mathematical difficulties*. For this reason, it was allocated as a single code in a grouping given the same name *advanced* to avoid confusion. The *mathematical difficulties* refer mainly to students from mathematics topics taught in Level 3 and Level 4 modules. Ten visits from students studying Level 2 modules are included as they relate to advanced areas in physics and economics as opposed to *mathematical difficulties*. The remaining two hundred and forty-two visits represented visits from 120 individual students registered to at least one of thirty-seven modules. Students from five modules showed exceptionally high attendance numbers.

The module that showed the highest number of visits in the advanced grouping although, not the highest in relation to the number of students registered to the class, was a Level 3 multivariable calculus module. The data showed there were seventy visits in total for this module from forty-one different students The percentage number of visits relative to the class size was thirty percent. One student visited on eight occasions, one on six, two on four, two on three, seven students visited twice and the balance of students attended on a single occasion. The number of students registered to this module was two hundred and thirty-one. This indicated that eighteen percent of the class attended the MSC during this eight-week period.

One Level three statistics module also showed high numbers of visits. Eighteen students attended the MSC. The percentage number of visits relative to the class size was also thirty percent. One student sought help on four occasions, one on three, five on two occasions and eleven students had a single visit, making a total of twenty-eight visit to the MSC for students from this module. The number of student registered to this module was ninety-four This indicated that nineteen percent of the class attended the MSC.

There were two Level 3 Complex Analysis modules that also showed relatively high numbers of their students attending the MSC. The data showed there were twenty-five visits in total for the module referred to as Complex Analysis A, from six different students. The percentage number of visits relative to the class size was sixty-eight percent. Six students attended with a very high number of visits, thirteen by a single student and another student attending on seven occasions. The number of students registered to this module was thirty-seven. This indicated that sixteen percent of the class attended the MSC. Seven students attending Complex

Analysis B module visited the centre on twenty-one occasions The percentage number of visits relative to the class size was fifty-three percent. One student visited on seven occasions and another two students attending five times. Thirty-nine students were registered to this module. This indicated that eighteen percent of the class attended the MSC during this eight-week period.

The fifth module from which high numbers of students attended was a Level 3 Financial Mathematics module. This Level 3 module showed the highest number of visits relative to class size, at seventy-four percent. Ten students from the class visited the MSC resulting in a total of twenty-three visits, one student visited on seven occasions and two students each attended five times. The number of students registered to this module was thirty-one. This meant over thirty-two percent of students registered to the class visited the MSC over this eight-week period.

Table 4.10 Details of modules with high attendance for the advanced code

Modules	*Class Size*	*Number of individual students*	*Total number of visits*	*Number of individual students as a % of class size*
Financial Mathematics	31	10	23	32%
Complex Analysis A	37	6	25	16%
Complex Analysis B	39	7	21	18%
Statistics	94	18	28	19%
Multivariable Calculus	231	41	70	18%

4.2.6 Grouping 6 – Other

All codes in this section other than *mathematical expressions* and modelling had 30 or less *mathematical difficulties* and the discussion of these codes is presented in an appendix (see Appendix C for further details of codes numbering less than 30). This grouping represented over 7% of the total *mathematical difficulties*.

Table 4.11 Other grouping and Codes

***Other* grouping**	**Number of *Mathematical Difficulties***
Mathematical expressions	*57*
Modelling	*39*
Sets	*23*
Co-ordinate geometry	*10*
Converting units	*6*
Pattern spotting	*2*
Total mathematical difficulties	***137***

4.2.6.1 Mathematical expressions

Difficulty with lack of understanding mathematical terms was evident from fifty-seven entries in the data relating to fifty-three student visits from twenty-two modules. The major term evident in the data was the understanding of the statistical term ‘variance’, this arose when not one of nineteen students attending a Hot Topic understood or could explain the relationship between variance and standard deviation. Other difficulties were recognizing symbols used for *partial differentiation*, summation, and sin(t) multiplied by sin(t) written as sin^2(t). Difficulty with expressions such as cardinality, identity, and discrete variable were also evident.

4.2.6.2 Modelling

The code *Modelling* is seen predominantly when students arrive at the MSC with "word problems" that students are required to convert into mathematics. Students taking twelve separate modules attended for assistance with this code. A typical example from a business module is as follows:

> *'Working on business problem where the profit had to be maximized or minimized. This involved taking derivatives but most of the difficulty was in the initial setup of the profit function.'*

Approximately one-third of *mathematical difficulties* in this code were exhibited by students from this business module.

This grouping completes the findings from the analysis of the *mathematical difficulties* exhibited by students attending the Maths Support Centre during the eight-week research period. The next section explores the general nature of the *mathematical difficulties.*

4.3 The Nature of Student Visits to the MSC

Many universities issue a mathematical diagnostic test early in the first semester to incoming Level 1 students. These can be beneficial in revealing deficits in prior knowledge and basic skills of the incoming cohort. In contrast, the data gathered in this research came from the *lived experience* of students attending a mathematics support centre over an eight-week period in the first semester 2014/2015 and aligns more specifically with the module content although, limited to those students who seek help.

Results of diagnostic testing, based on the whole cohort being examined, may show that students across the class are not good with inequalities or fractions, but these data recorded from the *lived experience* showed very few people came for help with these topics perhaps, because they were not part of their module content. Data from the *lived experience* was by its nature limited to those students who choose to visit the MSC. Attendance could have been caused by a range of different factors such as: the lecturer being conscientious and promoting the MSC to students on a module, the lecturer being very poor and the students coming to the MSC in desperation, students motivated by upcoming weekly worksheets or examinations, or perhaps, students seeking the highest grades.

This section outlines the general nature of the *lived experience* of students attending the MSC. Each student visit may relate to one or more *mathematical difficulties*. The results given below concern these *mathematical difficulties* and address the following research questions:

What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?

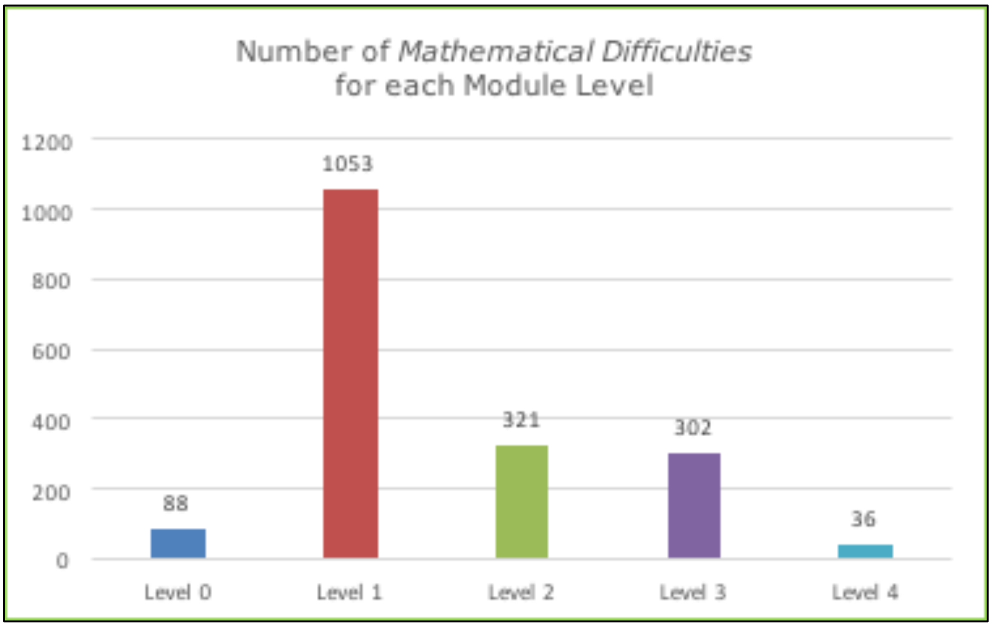
In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?

4.3.1 Mathematical Difficulties and Levels of Modules

The module and module level for which the student sought assistance, was recorded on the MSC database for each student visit and therefore the module relating to each *mathematical difficulty* can be identified. Figure 4.1 below displays the number of *mathematical difficulties* for each of the five module levels from Level 0 to Level 4, see Appendix B for a descriptor of these levels

Figure 4.1 below shows that the greatest demand for assistance was from students in Level 1 modules, at 58.5% and when Level 0 modules are added to this, the overall figure was 63%. These students were transitioning from post-primary education to third-level and this result would not be surprising as the greatest number of difficulties would be expected to occur at this level. However, Table 3.1 showing breakdown of student visits from 2009-2013 for the MSC, given in Chapter 3 showed the percentage of Level 1 visits had decreased over these four years and there was no reason to assume this trend would not continue. The percentages of *mathematical difficulties* for Level 2 to level 4 are approximately 18%, 17% and 2% respectively. Further analysis of the Level 1 *mathematical difficulties* shows that almost 23% of these were experienced by students specializing in mathematics. If the students taking engineering programmes are included, the percentage was over 41%.

Figure 4.1 The number of mathematical difficulties, displayed for each of the five module levels (Level 0 to Level 4)



For further analysis, the level of the module for which a student exhibited the *mathematical difficulty* is examined and these levels are displayed for each code in Table 4.12 below. Column 1 displays the list of codes (description of mathematical difficulties exhibited by the MSC students) and Column 2 the total number of *mathematical difficulties. Columns 3 - 7* inclusive provide the number of these entries for each of five levels – Level 0 to Level 4.

Table 4.12 Total mathematical difficulties displayed by module level

Code	Total Mathematical Difficulties	Mathematical Difficulties Level 0	Mathematical Difficulties Level 1	Mathematical Difficulties Level 2	Mathematical Difficulties Level 3	Mathematical Difficulties Level 4
Advanced	252	0	0	10	214	28
Discrete Mathematics	142	0	118	23	1	0
Vectors	142	3	102	32	5	0
Matrices	124	0	111	8	5	0
Mechanics	108	5	86	15	2	0
Continuous Distributions	108	0	35	69	1	3
Basic Algebra	89	14	60	14	1	0
Differentiation	71	0	60	7	4	0
Indices	65	16	43	5	1	0
Integration	64	0	40	7	17	0
Graphs	63	3	50	7	2	1
Partial Differentiation	58	0	0	56	2	0
Mathematical Expressions	57	3	40	8	5	1
Functions	48	6	32	7	2	1
Limits and Continuity	47	0	29	11	7	0
Trigonometry	45	4	39	1	1	0
Logs	43	1	35	1	6	0
Modelling	39	2	29	4	4	0
Discrete Distributions	36	0	19	13	2	2
ComplexNumbers	34	0	23	1	10	0
Fractions	26	11	14	1	0	0
Factorisation	24	4	16	1	3	0
Sets	23	0	12	7	4	0
Critical Points	22	0	22	0	0	0
Sign Rules	18	6	11	1	0	0
Inequalities	16	1	10	3	2	0
Domain and Range	11	2	2	6	1	0
Co-ordinate Geometry	10	1	8	1	0	0
Simultaneous Equations	7	3	4	0	0	0
Converting Units	6	3	1	2	0	0
Pattern Spotting	2	0	2	0	0	0

Closer inspection of Table 4.12 reveals some important results. *Vectors, discrete mathematics and matrices* display the highest levels of *mathematical difficulties* at Level 1. However, none of these are covered on the Leaving Certificate syllabi for mathematics. Mechanics also shows high numbers attending, this is not covered on the mathematics syllabi but is taken as a separate examination, named Applied Mathematics, for the Leaving Certificate and is available at both the Higher and the Ordinary Level.

What is noticeable is that the numbers attending for integration is fairly low at Level 1 and relatively high at Level 3. Few visits for difficulties with *integration* are shown at Level 2. The code *differentiation* also shows relatively high frequencies for Level 1 modules but low for other levels, see Table 4.13. But this is not surprising, as many first-year students must complete a calculus course. The majority of visits for differentiation are at this level, 51 of the 60 visits relate to two modules a business module and an Engineering module.

Table 4.13 Numbers of visits for differentiation and integration

	Differentiation	*Integration*
Level 1	*60*	*40*
Level 2	*7*	*7*
Level 3	*4*	*17*
Total	*71*	*64*

The content of statistics included on the Leaving Certificate mathematics syllabi has increased. It can be seen from Table 4.12 that the codes *continuous distribution* and *discrete distribution* show relatively low values at Level 1, which might be expected, but surprisingly high numbers at Level 2.

The high number of visits for the code *advanced* with approximately 85% of this code observed in Level 3 and 11% in Level 4 is perhaps unexpected but most students, taking

Level 3 or 4 modules that have a mathematics or statistics component, are likely to be enrolled on a degree programme that requires the study of mathematics to an advanced level. However, what is surprising is that approximately 10% of those coded in Level 3 and 9% in Level 4 were coded as very basic *mathematical difficulties* only and not also coded as *advanced*. The reason these are coded in this way is that the problem for which the student sought help, related to a basic *mathematical difficulty* and not with Level 3 or Level 4 module content. Below is an example to demonstrate this in the case of *complex numbers*:

> *'Decompose complex function into real and imaginary function. Didn't understand that a complex number 2+x+iy=(2+x)+iy and as such could not find conjugate to solve \frac{1}{2+x+iy} as a + ib. But student could find answer once given this information.'*

Mathematical difficulties seen in Figure 4.12 for the code of *graphs* at Level 1 are quite high and *partial differentiation* is noticeably high at Level 2. Lower levels of difficulty are seen in areas such as fractions, factorisation, critical points, sign rules and inequalities.

4.3.2 Prior Learning and Module Content

This section addresses the second research question:

> *What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?*

Table 3.1 *Breakdown of student visits from 2009-2013*, given in Chapter 3 showed the percentage of Level 1 visits had decreased over these four years and there was no reason to assume this trend would not continue. The high percentage of *mathematical difficulties* at Level 1 therefore, warranted further investigation. For example, were the *mathematical difficulties*, exhibited by students attending the MSC, related to content that was considered as a pre-requisite for the module, or related to content taught in the module itself? This is an important question. To examine this, each *mathematical difficulty* exhibited by a student in a Level 0 or 1 module, was classified according to whether the difficulty arose as either *Prior Learning* or *Module Content*:

i. *Prior Learning* – mathematical knowledge that is taken as pre-requisite knowledge for the given module,

ii. *Module Content* – mathematical knowledge taught in the module and not considered as pre-requisite knowledge.

The classifications above have only been considered where they relate to modules Level 0 and 1 for two reasons. Firstly, these were the modules that showed the greatest demand for assistance and secondly although, *Prior Learning* or *Module Content* could also be considered for the higher level modules, this would have necessitated both an understanding of which previous modules were taken by the students and also the content of these modules. These were not covered in the research study.

Figure 4.2 below displays the total number of mathematical difficulties at Level 0 and 1 for each code by classification of mathematical difficulty as Prior Learning or Module Content.

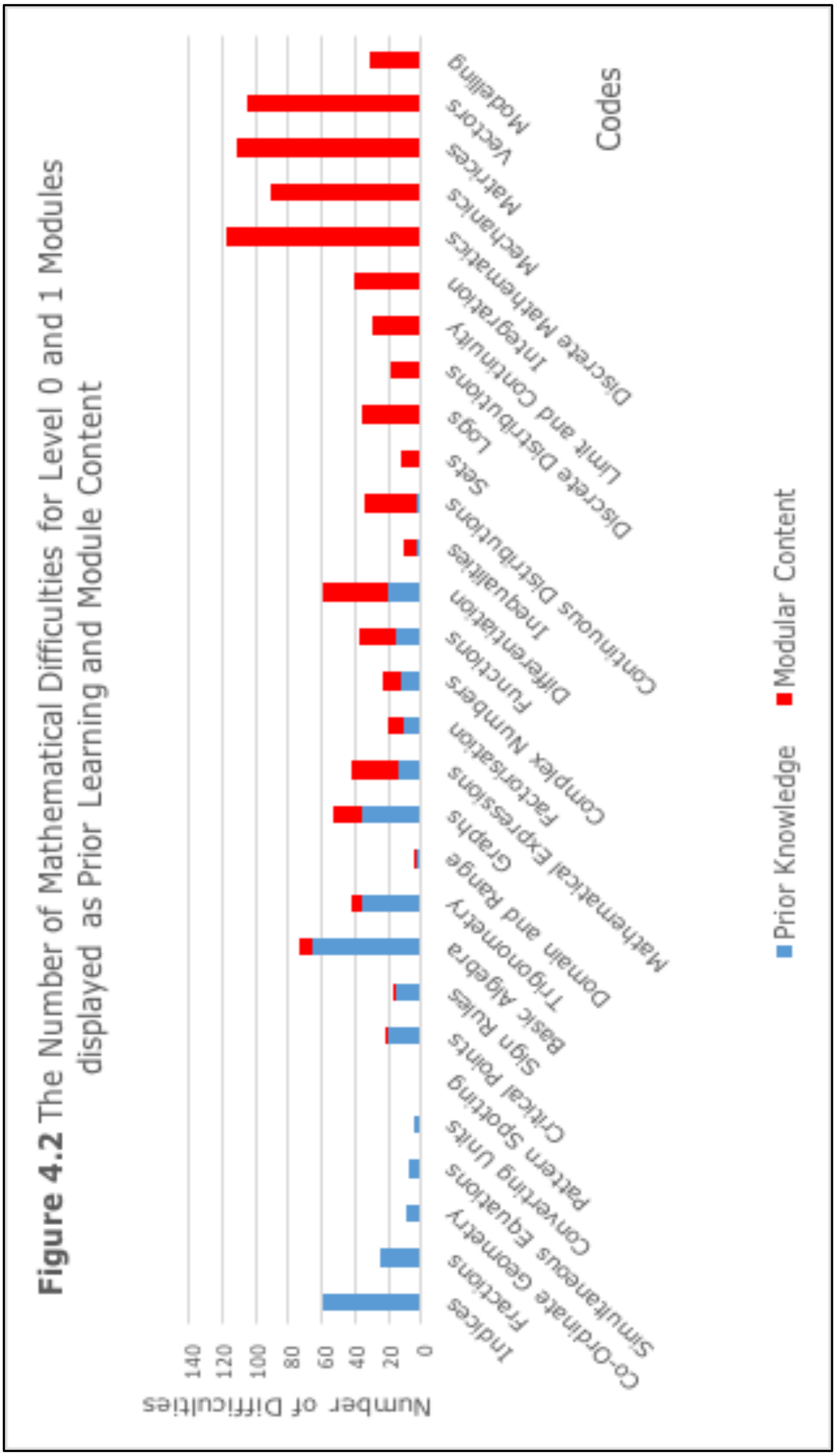


Figure 4.2 The Number of Mathematical Difficulties for Level 0 and 1 Modules displayed as Prior Learning and Module Content

The number of *mathematical difficulties* for *Prior Learning* is indicated in blue and the number for *Module Content is* indicated in red as seen in Figure 4.2 above. It is clearly evident from the histogram that the number of difficulties related to *Module Content* greatly exceed that due to *Prior Learning.* In fact, the analysis showed that the *mathematical difficulties* exhibited by students attending the MSC were more than twice as many for *Module Content* as were for *Prior Learning.*

However, there were a number of students who made high numbers of individual visits to the MSC. This can be seen in Figure 4.3 below. A check was therefore made to ascertain if these outliers had skewed the data presented in Figure 4.2. It can be seen from Figure 4.3 that the data were skewed to the right and that the graph levels off at five visits. For this reason, five visits by a student was then taken as the cut-off point and the data was further examined excluding any individual student who visited the MSC on more than five occasions.

Figure 4.3 Frequency of visits for individual students

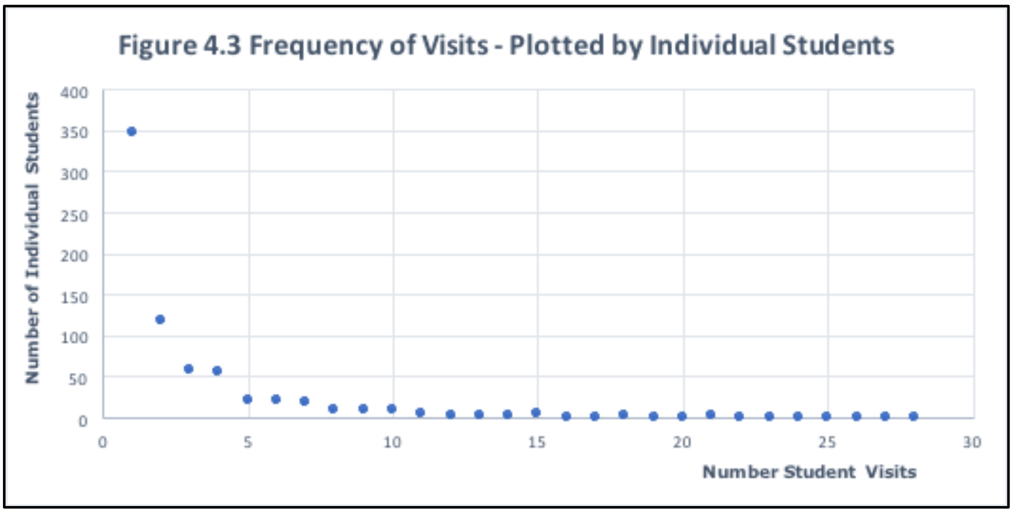
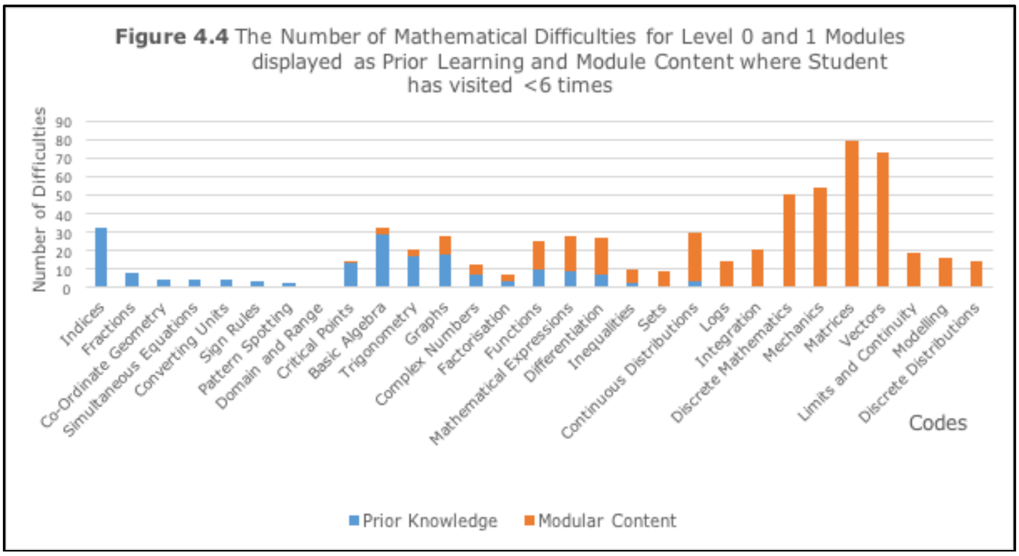


Figure 4.4 below shows the results of this analysis. Clearly, excluding the high attenders has not influenced the overall result in the relative numbers attending the MSC for *mathematical difficulties* in relation to *Prior Learning* or *Module Content* and therefore the conclusion that mathematical difficulties in relation to module content greatly exceeded that due Prior knowledge still held. This is a novel finding and was observed by exploring the *lived experience* of students visiting the MSC over the eight-week period of the research.

Figure 4.4 The number of mathematical difficulties for Level 0 and 1 modules displayed as Prior Learning and Module Content where a student has visited less than five times



The original objective of mathematics support centres was to provide effective support for students entering third-level education whose background in mathematics was found wanting (Hawkes & Savage, 2000). What can be observed from the above charts is the particularly high student attendance for *mathematical difficulties* in areas such as *discrete mathematics, matrices, vectors* and *mechanics*. None of these topics are covered in the new Leaving Certificate mathematics syllabi. These, among other topics, were treated as *Module content* in analysing the results. *Basic algebra* and *indices*, are the highest number of *mathematical difficulties* shown as *Prior Learning* but the number of visits in relation to these is shown to be much lower.

4.3.3 High attenders at the MSC

This section addresses the third research question:

> *In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?*

4.3.3.1 Use of Hot Topics for High Attenders at the MSC

One method of creating more efficient use of an MSC would be the introduction of *Hot Topic* sessions. In Semester 1 2014/2015 the MSC ran nine *hot topics*, five of which were held during the eight-week period of this research study. Four further *hot topic sessions* were organized after the completion of the research period in Semester 1. Each of the above nine sessions were organized for a unique module. To answer the third research question we have considered employment of the data to promote time efficiency. One method in which time efficiency can be improved is by concentrating teaching into group sessions. We consider how analysis of attendance by *mathematical difficulty* and module could help identify the source of student problems to identify areas suitable for Hot Topics.

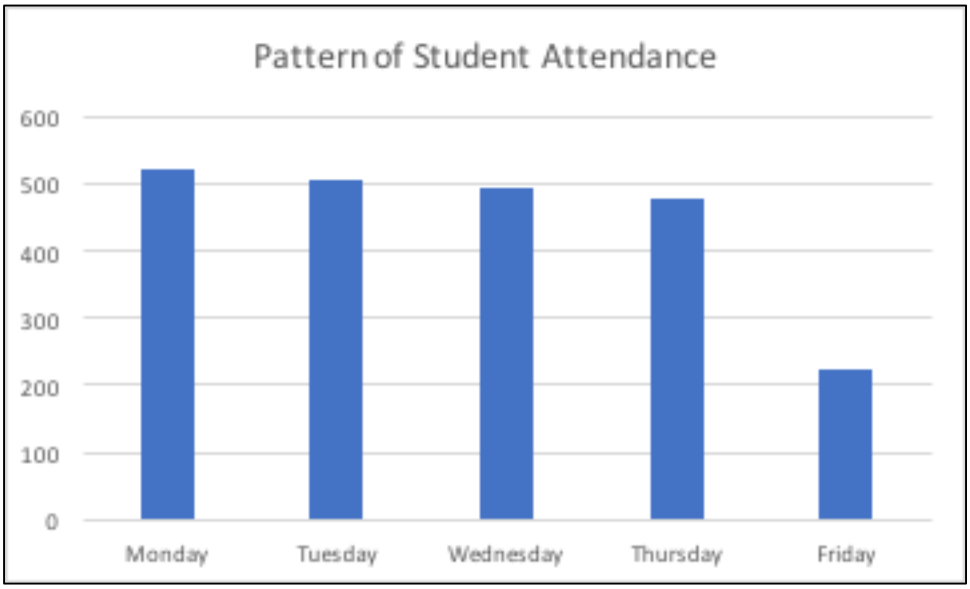
Table 4.14 below looks at all module/code combinations with greater than 10 visits for Level 1 students recorded over the eight-week period of the study. This information is potentially useful when identifying areas for *hot topics* and thus increasing the efficiency of the MSC by reduction in number of individual visits.

Table 4.14 List of potential modules for Hot Topics

Module Name (Anonymised)	Code	Total
Business Maths	Indices	28
Business Maths	Differentiation	27
Business Maths	Logs	22
Business Maths	Graphs	21
Business Maths	Modelling	14
Business Maths	Functions	13
Linear Algebra for Science	Matrices	60
Linear Algebra for Science	Vectors	18
Calculus for Engineering	Integration	27
Calculus for Engineering	Differentiation	24
Calculus for Engineering	Basic Algebra	17
Calculus for Engineering	Functions	10
Number Theory	Discrete Mathematics	76
Applied Maths for Engineering	Vectors	44
Applied Maths for Engineering	Mechanics	17
Applied Maths for Engineering	Trigonometry	12
Statistics for Business Studies	Statistics	29
Statistics for Business Studies	Mathematical Expressions	20
Statistics for Business Studies	Discrete Distributions	17
Applied Maths for Science	Mechanics	18
Applied Maths for Science	Graphs	14
Applied Maths for Science	Limits & Continuity	12
Applied Maths for Science	Critical Points	11
Linear Algebra for Maths Specialists	Matrices	31
Linear Algebra for Maths Specialists	Complex Numbers	18
Statistics for Science	Statistics	33
Statistics for Science	Discrete Distributions	10
Multivariable Calculus for Engineering	Partial Differentiation	37
Physics for Engineers	Mechanics	20
Physics for Engineers	Vectors	12
Combinatorics & Number Theory	Discrete Mathematics	27
Applied Biostatistics	Statistics	25
Applied Mathematics & Mechanics	Vectors	19
Access Mathematics Module	Basic Algebra	13
Calculus of Several Variables	Partial Differentiation	12
Foundations of Physics	Mechanics	12
Introduction to Mathematics	Indices	10
Mathematics for Agricultural Students	Basic Algebra	10
Mechanics	Mechanics	10

The weekday attendance pattern over the eight-week period is shown in Figure 4.5 below. Data looking at attendance by day of the week may be helpful in identifying times where *hot topics* could be considered.

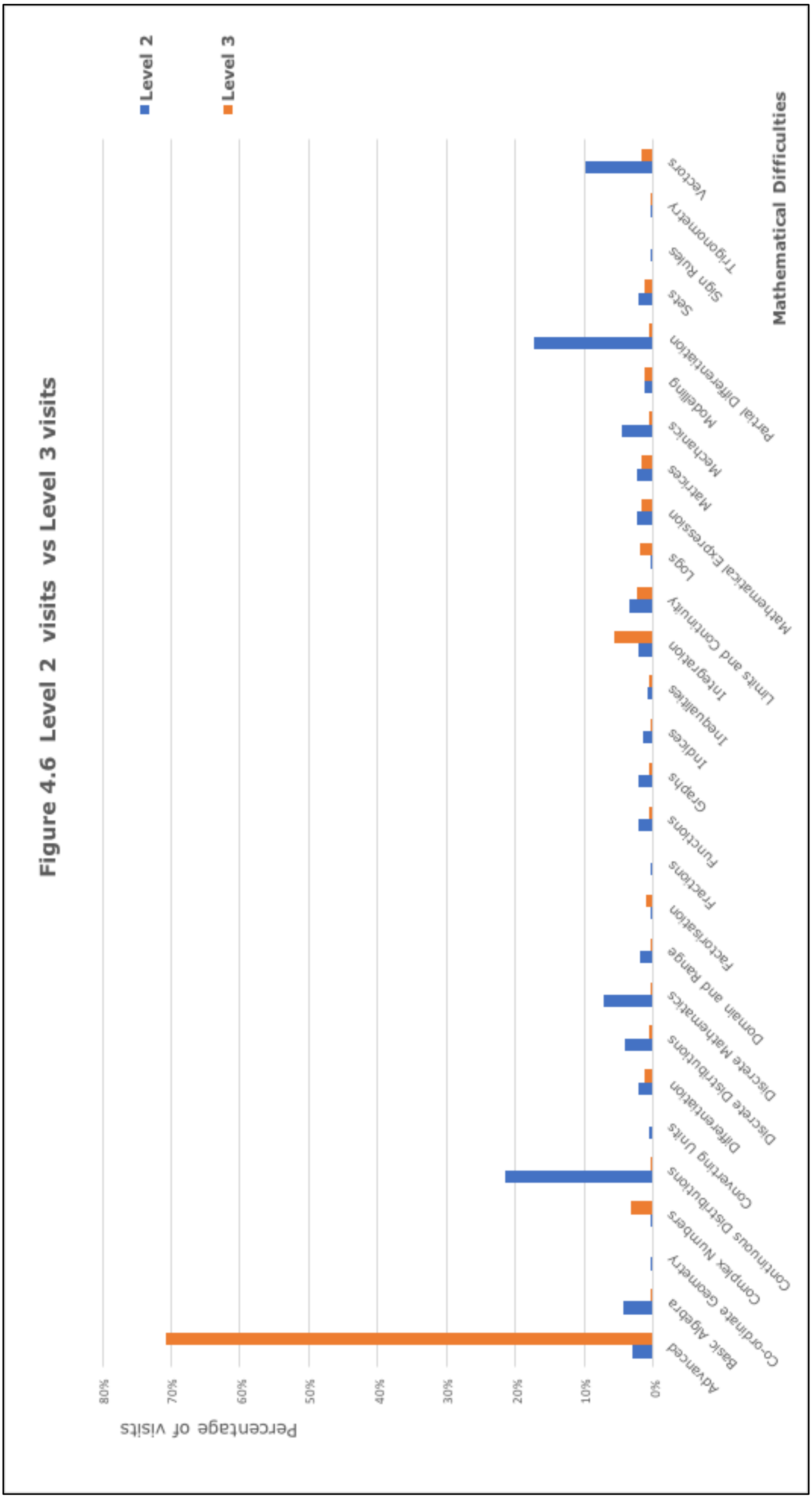
Figure 4.5 Attendance patterns at MSC over eight-week period



4.3.3.2 Prioritisation of access to the MSC based on attendance for each level

Figure 4.6 below shows the pattern of mathematical difficulties exhibited by students attending the MSC for assistance with Level 2 and Level 3 modules. What the figure indicates is that there were higher numbers of students attending the MSC in certain topics at Level 2 as seen for partial differentiation and statistics. What is evident for Level 3 is that there were very large numbers attending for one area, (topic shown as *advanced)* but the numbers were small for other areas of difficulty. However, the data coded as *advanced* were wide ranging in mathematical areas covered - 120 students registered to 37 modules see section 4.2.5 - and therefore not amenable to consideration for *Hot Topics*.

Figure 4.6 pattern of mathematical difficulties exhibited by students attending the MSC for assistance with Level 2 and Level 3 modules.



4.3.3.3 Mathematical difficulties by module

The study hypothesis was that analysis of data identifying modules with multiple problem areas (by number of *mathematical difficulties*) might aid in addressing efficiency of a MSC. *Hot topics* would not be time efficient if targeted at modules where only a few students attended for a small number of *mathematical difficulties*. The top quartile of visits per module occurs at greater than or equal to 6 visits, this top quartile might represent the minimum most useful target for *hot topics.* However, it also shows that only a minority of modules could potentially be targeted in this way (73/374, 20% see Table 4.15).

Figure 4.6 graphically represents the total number of modules per code as well as the proportion of modules with greater than or equal to six *mathematical difficulties*.

Figure 4.6 The top quartile of attendance by module (>=6 visits) compared to overall attendance

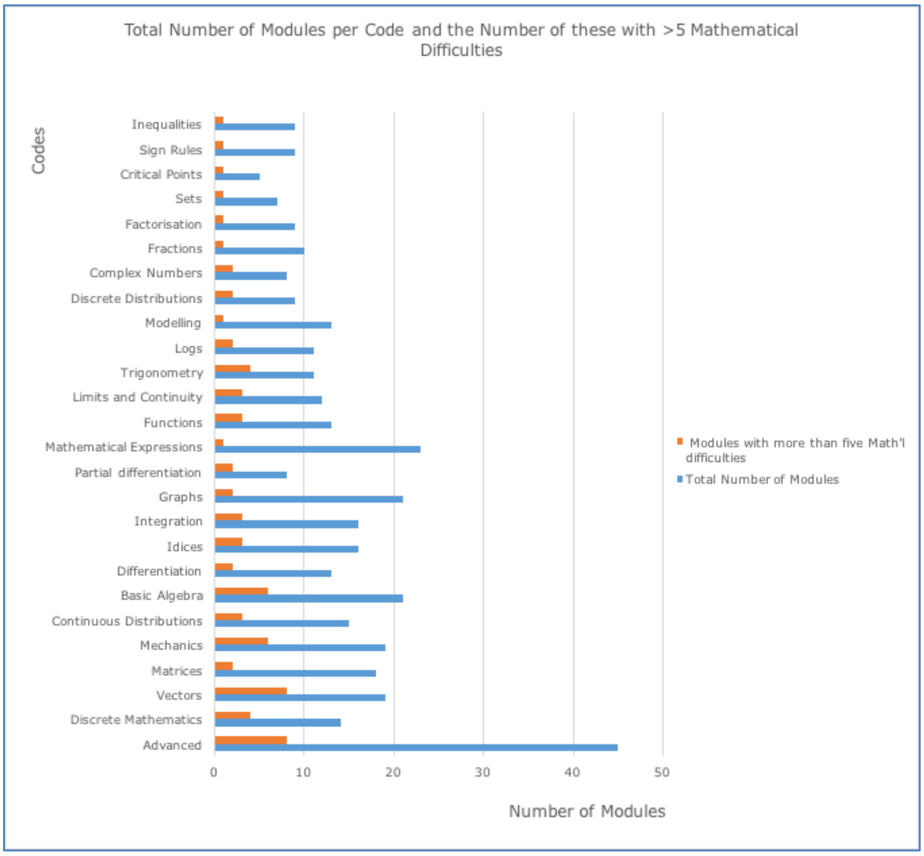


Table 4.15 Breakdown of mathematical difficulties by module

Code	Total Number of *mathematical difficulties*	Total Number of Modules	Modules with >=6 *mathematical difficulties (top quartile)*
Advanced	252	45	8
Discrete Mathematics	142	14	4
Vectors	142	19	8
Matrices	124	18	2
Mechanics	108	19	6
Continuous Distributions	108	15	3
Basic Algebra	89	21	6
Differentiation	71	13	2
Idices	65	16	3
Integration	64	16	3
Graphs	63	21	2
Partial differentiation	58	8	2
Mathematical Expressions	57	23	1
Functions	48	13	3
Limits and Continuity	47	12	3
Trigonometry	45	11	4
Logs	43	11	2
Modelling	39	13	1
Discrete Distributions	36	9	2
Complex Numbers	34	8	2
Fractions	26	10	1
Factorisation	24	9	1
Sets	23	7	1
Critical Points	22	5	1
Sign Rules	18	9	1
Inequalities	16	9	1
Total	**1764**	**374**	**73**

4.3.3.4 Identifying module problems; intervention with lecturers for modules with high attendance relative to class size

Data may also be used to provide feedback to lecturers on individual modules disproportionately represented at the MSC. Table 4.17 below presents student visits by module and class size.

Table 4.16 Individual modules (derived from Table 4.15) with highest number of mathematical difficulties

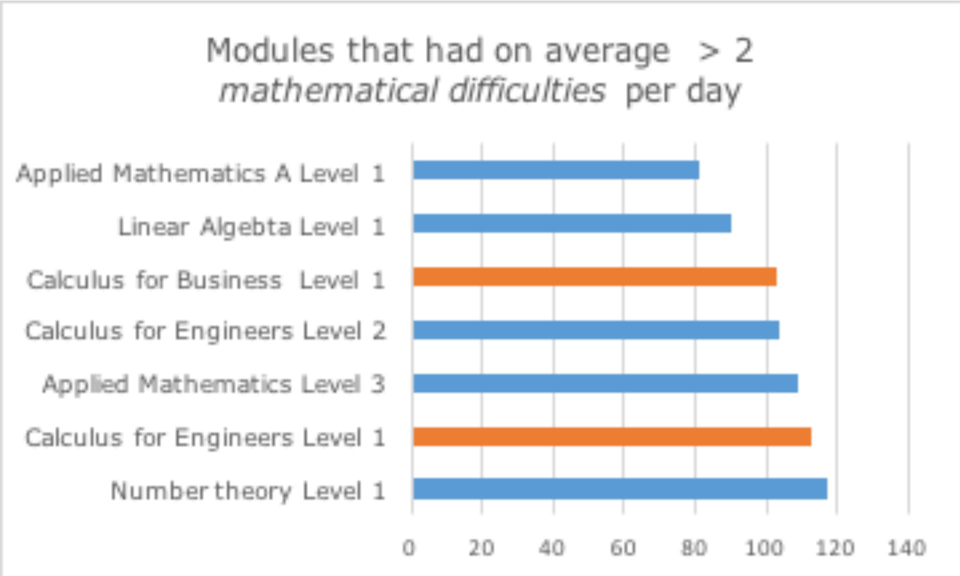
Code	Level of Module with Highest Number of *Mathematics Difficulties*	*Mathematical Difficulties* for Module with Highest Number of *mathematical difficulties*	Highest as % of Total Number of *Mathematical Difficulties*
Discrete Mathematics	Number Theory Level 1	76	54%
Advanced	Applied maths Level 3	69	27%
Matrices	Linear Algebra A Level 1	61	49%
Vectors	Applied maths Eng Level 1	46	32%
Partial differentiation	Calculus Eng Level 1	38	66%
Continuous Distributions	Statistics Level 2	37	34%
Integration	Calculus Eng Level 1	28	44%
Differentiation	Maths Business Level 1	27	38%
Idices	Maths Business Level 1	23	35%
Logs	Applied maths Eng Level 1	22	51%
Graphs	Maths Business Level 1	21	33%
Mechanics	Physics Level 1	20	19%
Mathematical Expressions	Statistics Level 1	20	35%
Basic Algebra	Calculus Eng Level 1	19	21%
Complex Numbers	Linear Algebra B Level 1	18	53%
Discrete Distributions	Statistics Level 1	17	47%
Modelling	Applied maths Eng Level 1	14	36%
Functions	Maths for Business Level 1	13	27%
Trigonometry	Applied maths Eng Level 1	13	29%
Limits and Continuity	Applied Maths Level 1	12	26%
Critical Points	Applied Maths Level 1	11	50%
Factorisation	Applied Maths Level 1	9	38%
Fractions	Introductory Calculus	7	27%
Inequalities	Maths for Computer Level 1	7	44%
Sets	Maths for Computer Level 1	6	26%
Sign Rules	Introductory Calculus	6	33%

Table 4.17 Modules with high number of student visits showing level of module, number of unique visits and percentage of class size

Module Name (Anonymised)	Level of Module	Unique Student MSC Visits	Class Size	% of Class Attending
Calculus for Engineering	Level 1	55	293	19%
Multivariable Calculus for Engineering	Level 3	51	231	22%
Applied Maths for Engineering	Level 1	49	293	17%
Number Theory	Level 1	48	114	42%
Linear Algebra for Science	Level 1	48	305	16%
Business Maths	Level 1	43	522	8%
Statistics for Business Studies	Level 1	41	519	8%
Linear Algebra for Maths Specialists	Level 1	34	186	18%
Applied Maths for Science	Level 1	32	65	49%
Statistics for Science	Level 2	30	221	14%
Physics for Engineers	Level 1	29	284	10%
Multivariable Calculus for Engineering	Level 2	24	281	9%
Calculus of Several Variables	Level 2	24	133	18%
Access Mathematics Module	Level 0	23	50	46%
Statistics Time Series	Level 3	20	94	21%
Applied Biostatistics	Level 2	20	293	7%
Combinatorics & Number Theory	Level 1	19	61	31%
Mathematics for Agricultural Students	Level 1	16	361	4%
Introduction to Mathematics	Level 0	16	87	18%
Applied Mathematics & Mechanics	Level 1	15	97	15%
Mechanics	Level 2	14	54	26%
Foundations of Physics	Level 1	13	297	4%
Vector Calculus	Level 2	12	67	18%
Advanced Linear Algebra	Level 3	11	30	37%
Linear Algebra	Level 2	11	45	24%
Financial Mathematics	Level 3	11	31	35%

Each student attending the MSC relating to a module is counted only once for that module regardless of the number of visits they made. This is to counteract the skewing effect of multiple individual visits on the data. Table 4.17 highlights modules that have a high number of students from a module attending the MSC. This is additionally described with reference to class size.

Figure 4.8 Modules that had on average more than 2 *mathematical difficulties* per day

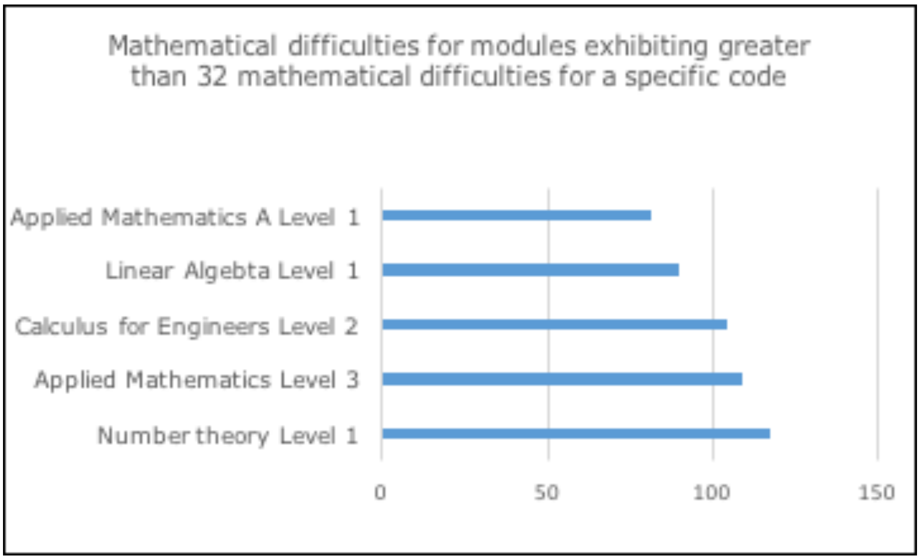


Students from certain modules show a high numbers of visits to the MSC by their students. Figure 4.8 illustrates the modules where these visits number more than 2 visits on average per day that is more than seventy-eight visits spread over the eight-week period.

Visits from students in five of these modules, as illustrated in Figure 4.9, related mainly to one or two specific codes (area of mathematical difficulty). This suggests the utilisation of *Hot Topics* for these modules may be a suitable method of reducing the overall number of individual visits to the MSC. Visits from students taking the two modules, Calculus for Business Level 1 and Calculus for Engineers Level 1, shown in red in Figure 4.8, related to a number of different codes with no individual code represented by greater than 32 visits.

However, *Hot Topics* may also prove beneficial for a number of codes exhibited in these modules.

Figure 4.9 *Mathematical difficulties* for modules with greater than 32 *mathematical difficulties* in at least one code



4.3.3.5 Continuous review of the data to enhance efficiency of data input; analysis of data from focus groups

A focus group was held in June 2015 with ten experienced MSC tutors who had tutored in both semesters in 2014-2015. The main aim of the focus group was to gain clarity and understanding of the feedback process from the tutors' perspective and explore with them means to improve the design and efficiency of the procedure.

An important point for the tutors was the time available for entering data:

'So I think it is important as well to actually say ... what should be our percentage in helping the student and helping the lecturer, is it eighty percent students, seventy percent, ninety percent?'

A further consideration was the recipient of the data:

'I think it depends on who is looking at it; if it is going to be [the researcher] or the lecturer . . . if it is the lecturer they might just want to know what the topic is, whereas [the researcher] might want more specific kind of feedback on what is affecting the [the student's] understanding.'

However, several tutors suggested that a lecturer might have reason to require more detail than just the topic as otherwise some important detail might be missed. The tutors gave the following example where students were studying a Level 1 differential calculus module in semester 1 but integration for these students was covered in the second semester:

'So obviously a lot of them had huge problems with the integration in the differential equations course, because they hadn't seen it before. … So I think it was important then to write, you know, that we are doing an integration question here and then (that) the student had not seen this before. So that is good feedback for the lecturer to see that.'

Also, if the entry is too brief then 'stuff that is around the course might not be captured,'

'It is the actual problem that I think is important for the lecturer as well. Because it is not enough to know, ok it is integration by parts, what is it about integration by parts that is causing the trouble?'

A tutor further suggested:

'... a good thing to have would be a separate list of the actual weakness that the reason this person was having problems was because … they have fundamental issues with fractions or whatever it happens to be, and I think that sometimes the information can get lost in the short paragraph that you write.'

While a drop-down option already existed at that time, '*it was a bit too general'.* One tutor proposed that a sidebar with a tree of topics and subtopics which could be selected and automatically added to the tutor entry would be helpful. Another suggested the following:

'I think the option of a tutor being able to add (to the drop-down list) a different topic that he sees coming up a lot, might be really useful.'

A combination of typing into a free textbox and using a pre-populated drop-down list to select topics which were frequently observed was suggested by a number of tutors.

'*Free text box always and then just dropdown to all the menu items and just tick and that is done.'*

During the eight-week research period tutors received frequent feedback on their *tutor entries* and the importance of this was mentioned by a number of tutors, one stating that '*we would need feedback on our feedback if we are going to make it better, '* and another,

'… we don't get feedback on our feedback at the moment. So it is very hard for us to … we are writing out what we have done, what we think was the problem or how to solve the problem. But we are not getting any details back about, you know, is this effective, is this a problem with the course, or you know, has this been resolved? We have no idea.'

In summary, the tutors wished for clarification on a number of issues. These related to the time involved in data entry versus tutoring, the intended recipient of the data and the level of detail required. They made a number of proposals for more efficient data entry, such as improved drop-down menus, availability of a free textbox and the ability to add a topic frequently requested by students. A difficulty noted by the tutors was where a student might wish to ask a very brief question in relation to a different module at the end of a session, this caused delay or incorrect module entry, as in the existing system the student had to log out and then log in for the new module. Finally, tutors emphasised the importance of feedback on their feedback to improve the quality of the data collected.

4.4 Interviews with lecturers

The aim of this section of the research was to assess lecturers' views on MSC feedback and answer the following question.

> *What feedback, if any, would be most beneficial for lecturers to receive on their students' visits to an MSC?*

This section describes the analysis of the interviews, focussing on six main questions as outlined below. When reporting lecturers' comments, the singular masculine pronoun has been used throughout to prevent any possibility of identifying a lecturer. An important aspect in assessing the validity of the data collected, was whether the lecturers would recognise the feedback as coming from their module and whether it 'made sense' to them. To that end the first question was:

> Did the lecturers recognise the tutor entries as arising from their module?

Lecturers, when presented with the tutor entries for their module, were asked if they recognised these as arising from difficulties with the mathematical content of their module. All lecturers said they did. This is how one lecturer answered the question.

> *'That is what they have done ... that makes sense ... it is all connected with the module, definitely.'*

Another lecturer commented that students from his module had sought help for some additional questions given on one of his worksheets:

> *'I recognise all the questions as they come from the assignments . . . So it was nice to see that some people are going to the Maths Support Centre with the unassessed problems.'*

A number of lecturers proposed, while reading the feedback, that a particular topic they observed in the feedback, frequently caused difficulty for students. One lecturer discussed the *mathematical difficulties* as he read the feedback, remarking that they were in line with expectations for the module:

> *'I can see that they would find that difficult.'*

This was the second question:

Were there instances, when the lecturer felt that the feedback related to a student's lack of pre-requisite knowledge for the module?

Seven individual lecturers made specific comments in relation to pre-requisite knowledge for their module. The final mathematics examination for the majority of students entering third-level in Ireland is the Leaving Certificate at either the Higher or the Ordinary Level. Some lecturers therefore, assumed that the basic topics have been covered. One lecturer, in response to the question of pre-requisite knowledge, stated that he believed that the difficulties arose with a small percent of international students who had not participated in the Irish system of education. Unfortunately, the MSC did not have access to data to investigate this further. Another lecturer, on reading the MSC tutor entries for their module, recognised areas covered at the Leaving Certificate, while another pointing to a number of entries, stated it was probably mathematics missed at post-primary level:

'[These] … are probably things that they missed in secondary school. Manipulating logarithms and so on and summing geometric progressions … I recognise that. I notice that a few people had trouble with manipulating the logarithms…'

A lecturer commented that this type of feedback might be useful to a novice lecturer or a lecturer teaching a module for the first time since the MSC feedback underlines the deficiencies in students' pre-requisite knowledge that such a lecturer might take for granted given they are covered on the post-primary syllabus:

'I think it is useful anyway . . . I can see that because it would sort of bring home to you the level of problems that students are having . . .

Certainly when you start out lecturing it is easy to assume that the stuff from the school syllabus is well known; which is obviously not the case.'

One lecturer suggested that a lecture was not a suitable environment to cover very basic algebra as students who are weak need more individual attention and those who are strong do not require it. Another lecturer noted that much of the feedback to him indicated that difficulties his students exhibited were not related to second-level mathematics and remarked:

'I didn't see anything like you know, adding fractions or basic things like that, because this is a course that doesn't really need that much from secondary school. We really use very little. I would assume other courses would need more.'

The third question was as follows:

Was the feedback from the MSC useful to the lecturer, if so, in what ways?

Responses to this question were quite varied but all lecturers confirmed that they found it useful. Lecturers frequently compared the usefulness, or otherwise, of the MSC feedback to alternative forms of feedback they received. One lecturer described why he found the MSC feedback more useful than other forms of feedback:

'It is like expert critique, you know, it is not just the student talking about what they are doing. It is somebody who has seen lots of stuff going 'Ok, this is the problem you have' and that is really valuable feedback for me.'

Other forms of feedback were alluded to by a number of respondents, in some cases they mentioned that they found it difficult to get feedback in lectures as students were reticent about speaking up. In other cases, they stated that if students were working hard and attending the MSC it was an indicator of real problems and this gave much better information on topics that a lecturer needed to focus on:

> *'It is actually much more useful to me because ... it tells me what they are actually having problems with. So the descriptions that I get from the Maths Support Centre, are probably better than the rest of that stuff put together.'*

A number of lecturers stated that they find the MSC feedback more focused on the specific *mathematical difficulties* than other forms of feedback they received. One lecturer rated it, as next in a measure of its usefulness, to information sought by students after lectures.

Knowledge of what parts of the module cause difficulty is important for lecturers to have, especially when there are large numbers in the module, as one lecturer commented:

> *'So, the place where maths support is really useful for me is probably from the lecture material; so if they are seeing something in the lectures and video that they just don't understand and totally not getting it and then go to you and say 'Look I just didn't get the . . . can you explain it to me?' That kind of stuff is useful to me. If, all of a sudden, ten people turn up there saying the same thing, [then] that is really useful to me.'*

Sometimes lecturers have taught the same module for two or more years and are very familiar with difficulties experienced by their students but some of these lecturers mentioned that they were still surprised by certain queries in the feedback where they expected the students would not experience difficulty. For example, a lecturer expressed surprise by students' problems with a specific topic this year that had not come to his attention the previous year.

> *'I wasn't aware of the vectors as being as big an issue as last year.'*

When looking at numbers attending the MSC, one lecturer remarked that if only a few students are seeking help with their module there is not much a lecturer can do about it. However, if the numbers attending are exceptionally high, then there is a serious issue and they would change the approach in their teaching.

For other lecturers the information they found most helpful was when a number of students sought help for the same topics. This allowed them then to spend more time on this topic. *Hot Topics* is an area some lecturers referred to when they spoke of a high number of students attending for assistance in a specific section of the module. They commented that they found this especially useful where students in large classes have a wide range of prior mathematics achievement. This is an example where a lecturer is referring to the benefit of *Hot Topics:*

> *'So if there was more engagement somehow between the class and what could be offered in the targeted way from the maths support centre, I think that would be great . . . something that might suit students who are lacking a given background, for example, . . . if I am teaching logs and exponents in first year I do this very quickly, I just do it in a couple of lectures, and most of the students have met that before but the students who haven't met it before struggle with it.'*

A lecturer remarked that he may receive information on a *mathematical difficulty* when one or two students discuss it with him or it comes to him through another forum, whereas, the fact that every problem, including what is covered in the *Hot Topic,* is documented and collated in the MSC feedback, is very useful.

> *'But the hot topic you need to kind of have the maths support centre there to have collected the area . . . (also) it was useful for me I would kind of know in advance these are the questions I am likely to be faced with, because these are the issues that are coming up. So I think that is very valuable form of feedback.'*

The importance of 'hard data' is emphasised by one lecturer as follows:

> *'It was very consistent with what we anecdotally had felt. So it is great that we now have the hard data gathered that says 'Look when they present with problems, these are the categories that they fall into' . . . you have to start with the data gathering… you know we are all kind of saying that we know that they have problems with this, that and the other, but if you are then going to justify developing any resources, it is great to have the quantitative piece behind it that says 'Look we have evidence that they are presenting with these things.'*

Finally, to investigate if the level of feedback to lecturers was too much or too little. This following question was posed to the lecturers:

> Was the level of detail suitable – too much/too little?

The key theme in the majority of lecturers' comments in answer to this question was *brevity* as they explained they had limited time to read the feedback and they queried if inputting such detailed entries was the best use of a tutor's time. However, one lecturer puts the answer to this question very succinctly as he remarked:

> *'Some people say it is too brief, some people say it is too detailed . . . you are never going to satisfy everyone on that front.'*

Some lecturers stated that the level of detail was suitable. One lecturer suggested that the *blue* comments, researcher additional details, for the first few weeks might be useful to target *Hot Topics.* Another lecturer advocated that 'not just the topic but the skill should be included'.

When discussing different forms of feedback, a lecturer proposed the level of detail was useful and stated:

> '(*The MSC feedback is*) *a formative kind of feedback as opposed to exams or the student feedback from the end of the semester, which is summative. So it is very helpful to have the formative feedback in real time as it were, so we can adjust things as the module goes on.'*

The usefulness of the feedback may be different depending on the module that is taught. This was implied by one lecturer who believed the detail was not worth the extra work but then reconsidered and implied it might be very different for him if he was teaching another module.

A number of proposals were made by the lecturers where they suggested the extra detail would be more useful for the MSC than the lecturer. The area of the course for which the student was seeking help, was described by one lecturer as the type of feedback most useful for him. This lecturer recommended, if the MSC had a list of topics for each module, it might be useful. Another lecturer agreed with the manager's proposal that a drop-down menu might make it easier for the tutors to enter data and it would be simple for the lecturer to give a list of topics. A further comment from a different lecturer advocated using the module descriptors as a menu for the tutors.

Did the opinion of lecturers vary between interviews?

There was little variation in the opinions of lecturers between interviews other than a few lecturers who commented at the third interview that the feedback might be more useful if the modules they were lecturing were different. A lecturer lecturing a Level 2 module suggested the feedback might be more advantageous if he was lecturing an introductory level mathematics module as he had in previous years. Another lecturer, lecturing an introductory module for a number of years, commented that he would probably find the feedback more valuable if he were lecturing a module for the first time. The majority of lecturers, other than stating that they found the information useful and giving reasons why they found it beneficial at the second interview their views were unchanged at the third interview.

Were there changes in lecturer practice as a result of learning from MSC feedback?

An important consideration when lecturers change their lecturing practice as a result of feedback from the MSC is that it may not alone assist those students who seek help in the MSC but may also prove beneficial for students who find the same difficulty but for some reason do not choose to come to the MSC.

How lecturers changed their practice as a result of what they learnt from the MS feedback is important. However, a limitation of the recorded data was that the specific question was not included in the original list of questions presented to the interviewer. Although, information arose fortuitously in a number of lecturers' comments.

Lecturers detailed few occasions where they had made actual adjustments to their programme during the semester. However, in one case a lecturer had noted students had difficulty reading statistics tables and had added an extra video online to assist the students. On observing previous feedback entered that semester and presented to him later on in the same semester, during the interviews, he suggested that these most probably arose prior to his uploading the video.

A different situation was mentioned by another lecturer where difficulties which had been recorded in the feedback, in particular from a *Hot Topic* was of benefit to him as it provided him in advance with answers to prepare for a revision class:

> *'These are the questions I am likely to be faced with, because these are the issues coming up.'*

Another lecturer, stated where he found the formative feedback from the MSC useful but did not describe exactly how he used it.

> *'So it is very helpful to have the formative feedback in real time, as it were, so we can adjust things as the module goes on. So I found that very useful.'*

A number of lecturers commented on changes they would make as a consequence of the feedback from the MSC. These were more suitable for implementation in the following year. For some, it was putting more emphasis on certain topics the following year:

> *' … this part for instance, is very useful, because next year I can tailor my topics according to their main difficulties (seen) here and spend more time on materials they found more difficult such as implicit differentiation … implicit differentiation is a topic that deserves more attention next year.'*

or altering the content of their course. This is how one lecturer expressed this:

> *'I just noticed that on the occasions that I have actually changed content, it has been more from a comment on the stuff on the maths support centre . . . I used to do a bit too much for them (the students) on sine and cosine and how you went from one quadrant to another … and I decided after feedback two years ago … to omit that section.'*

Another lecturer had noted from the feedback that his students exhibited difficulty with basic trigonometry which was required for his module covering vectors. The MSC had organised a *Hot Topic* that semester on *trigonometry* and the lecturer noted the importance of synchronisation of this with his lectures the following year:

> '… But I think it was definitely worthwhile doing that (*Hot Topic*), … I think I will try and synchronise these data next year.'

Overall these results suggest that the lecturers recognised the feedback as coming from their module and found the information useful. They noted instances, when they believed that the feedback related to a student's lack of pre-requisite knowledge for the module. There were a small number of instances where the lecturer stated they had changed practice or would do so the following year as a result of learning from MSC feedback. Their comments provide insights on a number of ways to improve the feedback. However, as the quote stated earlier, it would be difficult to please everyone.

Chapter 5 Discussion

Our findings in this section have demonstrated that the majority of *mathematical difficulties* were experienced by students in Level 1 modules. In the analysis, difficulties with *Module Content* were observed to exceed those for *Prior Learning.* Some modules indicated high numbers of *mathematical difficulties* for a specific code. Examining codes, where the number of *mathematical difficulties* was the highest for a single code, the respective modules examined did not yield results showing exceptionally high daily attendances. However, when *mathematical difficulties* were counted across all codes, the total number of these difficulties was shown to be more significant. Lecturers found the feedback beneficial but stated brevity in the feedback was important for them.

The UCD Maths Support Centre (MSC), established in 2004, presently offers a free, drop-in service in mathematics and statistics support to all UCD students registered to a Level 0, 1 or 2 module, irrespective of their programme of study. It is a very busy centre with, on average, 5,500 visits per annum over the last three years. Feedback, generated at the UCD MSC on students' visits, is accessible to all lecturers within the School of Mathematics and Statistics in real time (Cronin & Meehan, 2015). This provides the individual lecturer with information relating to: the number of students who visited the MSC with a mathematical query in relation to the module; the length of the visit; and the nature of the query. Further information on the UCD MSC, during the period of this study, is available in the Annual

Report of the academic year 2013-2014 (available at www.ucd.ie/msc).

5.1 Mathematical difficulties

A number of reports in both Ireland and the UK describe what has been commonly known as the '*mathematics problem'* (Howson et al., 1995; O'Donoghue, 1999; State Examinations Commission, 2000; Sutherland & Pozzi, 1995b). By the end of the first decade of the 21st century the mathematical under-preparedness of students entering third-level education had been well documented (Gill & O'Donoghue, 2006a; Lawson, 2003; Ní Fhloinn, 2009a). The National Council for Curriculum and Assessment (NCCA) discussion paper on post-primary education (2005) in Ireland, the Vorderman Report (Vorderman, Porkess, Budd, Dunne, & Rahman-Hart, 2011) and Smith Report (2017) in the UK all made recommendations for the reform of post-primary mathematics education.

The requirement for some form of support for students in the transition to third-level was recommended by the Smith Report (2004). Mathematics Support Centres have been seen as one option for providing this support and were mainly introduced to provide mathematical support to students in the transition from post-primary to higher education (Croft & Grove, 2006; Lawson, Halpin, & Croft, 2003; Mac an Bhaird, Morgan, & O'Shea, 2010).

Measurement of the decline in the mathematical skills of incoming students to third-level has been evidenced in reports from longitudinal studies of diagnostic tests (Gill &

O'Donoghue, 2006b; Lawson, 2003; Ní Fhloinn, 2009a; Treacy & Faulkner, 2015). In a foreword to a booklet describing various case studies of diagnostic testing (LTSN, 2003), Lawson suggested that, faced with changes in the homogeneity in the background of students entering third-level education, many institutions had introduced some form of diagnostic testing. The main aim of these tests, he observed, was twofold - firstly to inform staff of the mathematical competence of their incoming students; and, secondly to inform students of the outcome of the test with a view to remedying any shortfalls in their mathematical skills. Diagnostic testing is particularly beneficial as it supplies information for the total cohort tested. But the importance of adequate follow-up support following diagnostic testing is essential for success of the process (Lawson, Croft, & Halpin, 2003). Ni Fhloinn, Mac an Bhaird, & Nolan (2014) noted that some students may not see the link between diagnostic testing and their module content

The results in the present study, on the other hand, came from the *lived experience* of students seeking assistance in the MSC and although it is limited to those students who chose to come to the MSC it provides a picture of *mathematical difficulties* related to third-level education and not limited to basic skills. Students may at times not realise their difficulty relates to prior knowledge but receive immediate help from the tutor in these cases. Many lecturers encourage students to attend the MSC and indeed during the semester students may be motivated to seek targeted help at the MSC by, for example, poor marks in a mid-term assessment as suggested by Kulesagaram, 'motivation to succeed on assessment has long been recognized as a prime factor in student behaviour: assessment drives learning,' (Kulasegaram & Rangachari 2018, p.6).

Both methods of obtaining information on students' *mathematical difficulties* may be of benefit as qualifications of those entering third-level education diversify over time with for example, increasing numbers of international students registering for university programmes. The *lived experience,* combined with reporting back to the lecturer, may be of particular importance in updating lecturers where syllabi for entrance level examinations may be revised as seen recently in both Ireland and the UK (Lubienski, 2011; Smith, 2017).

Data recording the day-to-day work of tutoring in an MSC, and made available to the module lecturer, may also help faculty develop a deeper understanding of these *mathematical difficulties*, pre-requisite or module, and where they arise in a particular cohort. This could allow for the provision of targeted help for these students such as through the provision of *Hot Topics* or alert the MSC and lecturer to more significant student *mathematical difficulties* in a specific module.

Reports such as those by Gill and O'Donoghue (2007b), Ní Fhloinn (2009a) and Hunt and Lawson (1996) provided verification of these difficulties by employing the results of diagnostic testing. Evidence of *mathematical difficulties* was also previously obtained by way of questionnaires or surveys issued to students and lecturers which requested, amongst other information, details of the *mathematical difficulties* experienced by students in third-level institutions. For example, Lawson, Halpin and Croft (2003) in a survey, which looked at the number and role of MSCs, asked staff which topics were most commonly requested by students requiring assistance in the MSC. Other reports such as Sheridan (2013) simply mentioned general areas of mathematics, such as algebra and arithmetic, causing difficulty for students.

Diagnostic testing is not conducted with Level 1 students in UCD. This may be due to the large and diverse student cohort entering a range of programmes. Even if it were, it would be challenging to provide follow-up support. Lawson, Croft, and Halpin (2003) noted that a lack in the provision of follow-up had been suggested as a shortcoming of the process of diagnostic testing. The MSC in UCD has, for a number of years, suggested that documenting the *mathematical difficulties* exhibited by students in the *lived experience* of attending the MSC and simultaneously informing lecturers of these issues (Curley & Meehan, 2011) is another form of assisting students in making the transition to third-level.

The present research focused on identifying, recording and analysing areas of *mathematical difficulty* that students encountered while working with tutors in the centre.

As outlined in Chapter 4, these *difficulties* were classified into six categories:

- *Algebra;*
- *Calculus;*
- *Applied Mathematics;*
- *Statistics;*

- *Advanced;* and
- *Other.*

The research showed that in many cases students had issues with basic mathematical skills. By basic mathematical skills, is meant skills that the majority of students might be expected to have mastered for the Irish Junior Certificate or the Ordinary Level Leaving Certificate Mathematics examination. For the first category of *algebra* these difficulties were demonstrated in areas represented by codes such as *basic algebra*, *factorisation*, *indices* and *fractions.*

When basic mathematical skills were considered, the findings of this study were broadly in line with results reported in other studies. This study showed the code of *basic algebra* had the highest number of these difficulties. This finding was supported by Lawson, Croft, and Halpin (2003) and Sheridan (2013) The Chief Examiner's report (State Examination Commission, 2005) stated that in examination of the Higher Level Leaving Certificate Mathematics examination papers that deficiencies were evident in algebra, and that for the Ordinary Level mathematics examination papers:

> '*Average candidates experience difficulty with all but the most basic of algebraic manipulations and can cope only with basic routines in solving equations. Usually they fail to exploit quicker solutions than those offered by the practiced routine* (p.49).'

The majority of the Level 1 students involved in this research would have done their Leaving Certificate mathematics examinations in June 2014 and many of the *mathematical difficulties* in this research related to students whose minimum entry level requirements for these modules would have been the Ordinary Leaving Certificate in mathematics.

The Chief Examiner's report in 2015 (State Examinations Commission), is the only CER since the implimentation of *Project Maths* and is therefore relevant when observing the effects of the new approach to mathematics teaching at secondary level. The Chief Examiner provided information on changes which occurred in numbers taking the various levels of the Leaving Certificate examinations, the grades awarded and the variation in the gender of students taking the examinations over the previous five years. He also discussed the purpose and content of the revised syllabi. The information on the actual performance of students referred to questions asked in 2015 and although not specific to 2014, provided evidence of the benefits or otherwise of the new approach adopted since 2011. For example, the chief examiner found that the overall ability to accurately apply basic skills was lacking for some candidates taking the Higher level paper and this had increased since 2011. A further cause of concern for the examiner was that many candidates at Ordinary level exhibited a lack of knowledge of standard procedures and a lack of basic competence in algebra.

The *mathematical difficulties* with *basic algebra,* evident in this research were mainly exhibited by students in three modules each requiring an Ordinary Level Leaving Certificate in mathematics and would compare with the CER (2015) findings. One of these modules was a small introductory mathematics class, with 50 students, and the percentage of difficulties compared to the class size, at 26%, would be high but expected at this level. However, students with higher levels of mathematics requirement for entry to the programme also exhibited difficulty in this area. These mainly included students from one very large engineering module exhibiting half of the difficulties recorded for students at the higher level of entry requirement. However, when the number of students, almost 300 taking this module, was considered the percentage difficulty, at 6% was low also showing consistency with the (2015) report.

Indices, and in particular fractional and negative powers, were significant issues for students tested in Gill and O'Donoghue (2007b), Ní Fhloinn (2009a), and Hunt and Lawson (1996). Findings, in this study, agreed with these reports and identified this area as the second highest in the numerical order of mathematical skills causing difficulty for the Level 0 and Level 1 MSC students, see Figure 4.2.

Inequalities were demonstrated in the results of diagnostic tests by Gill and O'Donoghue (2006b) as a cause of significant difficulty. Students from nine different modules in this study exhibited *mathematical difficulties* with *inequalities.* However, only 16 visits in total were recorded for these modules over the eight-week period. Results of diagnostic testing, based on the whole cohort being examined on the other hand, may show that students across the class are not good with inequalities as seen above. But data recorded from the *lived experience* of students attending the MSC showed very few people came for help with these topics. A probable reason for this is that *inequalities* were not covered in the lecture syllabi and were therefore, rarely observed. This is a novel finding and arose as a result of the research methodology which observed the *lived experience* of students attending a university mathematics support centre.

High numbers of visits were seen for *vectors* and *matrices.* This is possibly explained by the removal of these topics from the new Leaving Certificate mathematics syllabi. One lecturer commented in the interview with the lecturers that he noticed students had greater difficulty with *vectors* that year than in previous years.

Mechanics is not included in the Leaving Certificate mathematics syllabi but is available as a separate subject known as Applied Mathematics. This is available at both the Higher and the Ordinary Level Leaving Certificate. A very small percentage, approximately 4% (SEC, 2015) took this examination in the year prior to the eight-week data collection and it was not a requirement for any programme in UCD. Therefore, difficulties in this topic might be expected.

Logarithms are not included in the Ordinary Level Leaving Certificate mathematics syllabus and 22 out of 35 Level 1 visitors, looking for help with this topic, were from a single module which had a minimum mathematics entry requirement of an 02, see Table 2.1. It was interesting that a lecturer, looking at the feedback, commented that a number of his students had difficulty with logarithms. The minimum requirement for his module was the Ordinary Level in mathematics so perhaps he was not aware that logarithms were not covered at that level.

What was noticeable from results of recording the *lived experience* of students attending the MSC was that *differentiation* had a slightly greater number of *mathematical difficulties* when compared to *integration*. The code *differentiation* showed relatively high frequencies for Level 1 modules but low for other levels, see Table 4.13. But this is not surprising as many first-year students must complete a calculus course. Also, the study of the practical applications of differentiation has increased at the Ordinary Level Leaving Certificate, however, the skills such as the use of product rule, quotient and chain rule are not covered. Coverage of integration has decreased in the Leaving Certificate syllabus at the Higher Level and is not covered on the Ordinary Level. The numbers attending for assistance with integration were low at Level 1 and relatively high at Level 3. Few visits for difficulties with *integration* are shown at Level 2. When mathematical complexity is considered, the number of *mathematical difficulties* for integration might be expected to be higher than that shown for *differentiation*. This was not evidenced in the *lived experience* of students attending the MSC. The data was further analysed to establish the reason for this, and showed the majority of visits for *differentiation* are at Level 1. At this level, 51 of the 60 visits related to two modules (a business module and an engineering module). The students in the engineering module also study integration in their Level 1 module. For this engineering module, all those students who sought help with *differentiation* also sought help with *integration*. However, the 24 students from the business module do not study *integration*, hence explaining the higher numbers for *differentiation*. Students in the engineering module discussed go on to study *partial differentiation* in their second year of study and this was coded separately as previously explained and makes clear the higher number of difficulties with *partial differentiation* and lower value for *differentiation* as seen at Level 2, see Table 4.13.

The content of statistics included on the Leaving Certificate syllabus for mathematics has increased. The codes *continuous distributions* and *discrete distributions* show relatively low values, as therefore, expected, of *mathematical difficulties* for students in Level 1 modules since the increase in statistics in the new Leaving Certificate mathematics syllabi. But they display surprisingly high numbers at Level 2. However, these high numbers may be explained by two Level 2 modules teaching statistics at the introductory level to two large classes. These students would not have covered a module in statistics since their Leaving Certificate mathematics examination two years previously and the requirement for their programme would have been at the Ordinary Level Leaving Certificate mathematics examination. Also the full new programme for the new mathematics may not have been covered in some schools at that time So both the extended time since they had done any statistics, the new syllabus with the increase in statistics covered for the Leaving Certificate not fully implemented the year the majority did their Leaving Certificate, and the lower level requirement might have contributed to the high numbers shown here.

From observing entries in Table 4.13 for Level 2 modules, the statistics modules seen above, displayed the highest number of *mathematical difficulties*, the other main area of difficulty at this level, was for *partial differentiation. Partial differentiation* is normally introduced for the first time in UCD at Level 2 so the difficulty, as seen in Table 4.12, would not be unexpected. *Vectors* and *discrete mathematics* were seen to exhibit much lower incidence of *mathematical difficulties* at Level 2. This is possibly due to fewer students studying these topics in Level 2 programmes and also the coverage of these areas would be a smaller element of the course content for these modules.

Sketching graphs, both linear and quadratic, and understanding functions, were evident as basic difficulties experienced by students in the *calculus* category. This was in agreement with previous research by Ní Fhloinn (2009a) and Gill and O'Donoghue (2006b). Basic differentiation was not an issue in the present study. However, differentiation using product, quotient and composite functions was shown to present a significant difficulty for students. This finding was similar to that of Gill and O'Donoghue (2007b). Not all diagnostic tests included differentiation as a topic to be tested. The results of this research and the following remark by Ni Fhloinn (2009a) suggest that this may be a cause for concern. The author noted that 'there is a real danger that the diagnostic test currently being used was not suitable for the more demanding modules' (2009a, p.373). The advantage of maintaining the same diagnostic test over a number of years is that it allows long term analysis to be carried out. So the suggestion of the use of two tests with one a subset of the other by Ni Fhloinn (2009a) may be a practical solution particularly, where syllabi for second-level examinations are changed as seen in Ireland and the UK in recent years.

The *applied mathematics* category showed difficulty with basic trigonometry. This was mainly evident in three modules which is surprising as many of the students studying these modules have an entry level equivalent to that required for a Category C module (see Table 4.1). Problems with basic trigonometry are not universally evident in the literature. In one study, it was not tested (Ní Fhloinn, 2009a); in another it was not shown as significant (Haßler, Atkinson, Quinney, & Barry, 2004). It was shown, however, by Gill and O'Donoghue (2007b) as an issue for a number of students. The Chief Examiner's Report (2005) found that in the case of trigonometry '*good candidates do very well and weak candidates do very badly*' (p.39). This may offer an explanation for the finding in this study. Also, the Chief Examiner's Report (2015) showed that marks for trigonometry ranked low in overall examinations marks for the Ordinary Level examination of that year and on the lower half of the ranking in the Higher Level.

The main findings, in the *statistics* category, indicate students experienced basic difficulties with normal distributions, t-distributions and reading statistical tables. This is evident across the three key modules in which students experience difficulty. The new syllabi for Leaving Certificate Mathematics have an increase in the proportion of the syllabi dealing with statistics and probability. However, reading normal distribution tables is introduced in the Higher Level Leaving Certificate syllabus and is not included for the Ordinary Level. The requirement level for entry to these modules did not require a Higher Level Leaving Certificate and as such it might be expected that this could be an issue.

Students taking a number of specialist Level 2 modules, included in the *advanced* group, seek help in the MSC and as Lawson, Croft and Halpin (2003) state '*the people who staff a centre are undoubtedly the key resource and are highly influential in the success (or otherwise) of the centre'* (2003, p.13). To cater for these students, an MSC requires high quality tutors with high levels of mathematical and statistical content knowledge.

The majority of *mathematical difficulties* exhibited by students taking higher level modules are for the code *advanced* with approximately 85% of this code observed in Level 3 and 11% in Level 4. This is not unexpected as most students, taking Level 3 or 4 modules that have a mathematics or statistics component, are likely to be enrolled on a degree programme that requires the study of mathematics to an advanced level. However, what is surprising is that approximately 10% of those coded in Level 3 and 9% in Level 4 were coded as very basic *mathematical difficulties* only and not also coded as *advanced*. The reason was that the problem related to a basic *mathematical difficulty* and not with Level 3 or Level 4 module content.
Students studying for the Higher Diploma in mathematics may take a number of Level 3 and Level 4 modules. So a possible explanation for the elementary errors exhibited by students here may be that the students studying these modules may have been taking a Higher Diploma in Mathematics programme and might therefore be coming back after a study break and have forgotten some basic mathematics. There is also another possible reason when pass marks are set at 40%, students are not sufficiently competent in the mathematics covered at lower levels and gaps resulting from lack of previous knowledge becomes evident. Unfortunately, the research data did not allow further checking of the data to confirm or deny this reasoning.

The *mathematical difficulties* that arise in the *other* category are relatively few and therefore are easily handled within the everyday MSC operation.

Two recent studies extend our knowledge of students' *mathematical difficulties* and we discuss their findings relative to those in this study.

The first study was an account of a survey completed by 460 students studying in four separate third-level institutions in Ireland by Ní Shé, Mac an Bhaird, Ní Fhloinn, and O'Shea (2017). The second study by Carr, Murphy, Bowe and Ní Fhloinn (2013) investigated results of a mathematics diagnostic test given to third-year students. This mathematics diagnostic test covered many of the important concepts these students had studied in earlier years and was therefore also useful as comparison to this study.

In the first study two of the institutions surveyed were universities and two were Institutes of Technology. Institutes of Technology (ITs) are further and higher education colleges. There are currently fourteen ITs in Ireland. Despite their titles, they are not confined to studies in technology, and engage in both teaching and research in a wide range of disciplines, much of which is at university level. The first study found that 64% percent of responding students had taken the Higher Level Leaving Certificate mathematics examination, 32% had passed the Ordinary Level mathematics examination and 4% had either the Foundation Level mathematics or the mathematics qualification was not provided. The further breakdown by Leaving Certificate grade of these examinations was not provided in the study. The authors mentioned that the requirements for entry to universities are normally higher than for Institutes of Technology. At the time of completion of the survey, the majority of the students, who were registered to a number of different programmes, were finishing their Level 1 studies with a small number finishing Level 2. In the survey completed in Spring 2015, students were asked in open-ended questions, which topics caused them the most difficulty and which of the two, ideas or methods, had caused the greater issues.

The students named the following topics in order of decreasing difficulty: integration; differentiation; functions and graphs; logs and indices; and limits. A small number of students mentioned matrices, vectors and algebra. A limitation of these results was reported by the authors where '*most of the "easiest topics" identified by students were also identified as "topics causing difficulty" by other students'* (Ní Shé, Mac an Bhaird, Ní Fhloinn, O' Shea, 2017, p. 9).

However, results in these findings run counter to the results of that study, and showed that the highest numbers of *mathematical difficulties* related to issues experienced by students when working with *vectors*, *matrices* and *basic algebra.* Indeed, difficulties with *indices* were more numerous than those associated with *functions* or *graphs*.

The code *differentiation* also showed relatively high frequencies for Level 1 modules. But this is not surprising, as many first-year students must complete a calculus course. The study of the practical applications of differentiation has increased at the Ordinary Level Leaving Certificate, however, the skills such as the use of product rule, quotient and chain rule are not covered. Coverage of integration has decreased in the Leaving Certificate syllabus at the Higher Level and is not covered on the Ordinary Level. The number of *mathematical difficulties* for *integration* might be expected to be higher than that shown for *differentiation;* however, the data show the number of *mathematical difficulties* for *differentiation* are slightly greater than for *Integration.* The data were further analysed as seen in the results section (see Table 4.13), and earlier explanation. Also the students in the Engineering module study *partial differentiation* at Level 2 and since visits by students for this code are listed separately it explains the low numbers for *differentiation* at Level 2 but high numbers for *partial differentiation.*

The findings of the diagnostic test (Carr, Murphy, Bowe, & Ní Fhloinn, 2013), in the second study taken by students entering their third-year in the Institute of Technology and referred to above suggest that the students had struggled in all areas tested except in cases of very basic applications of differentiation and integration. Questions on matrices and complex numbers were attempted in the diagnostic test by a very small number of students indicating students had major difficulty in these areas. A similar result was evident in this study with students finding difficulty at Level 3 with complex numbers. These were mainly exhibited by students studying a module covering complex analysis. *Matrices* were not studied at Level 3 at the time of this study and the number of difficulties at this level were few and arose only as incidental to other modules being studied. The (2013) study discussed above indicated that *integration* had slightly higher levels of difficulty for the students when compared with results for *differentiation*. The present study showed considerably higher difficulty with *integration* at Level 3 (see Table 4.13) when compared to *differentiation*. This was probably a more typical result as a number of lecturers would regard integration as the more difficult topic.

Mathematical difficulties, where basic mathematical skills are not the principal issues, were evident in this study and this is discussed further in response to the following question.

> *What do these mathematical difficulties reveal about the nature of students' visits to the Maths Support Centre? Specifically, what proportion of visits relate to difficulties experienced with module content as opposed to lack of (prerequisite) prior knowledge?*

The approach taken in this study was unlike previous work done in this area. The data, gathered from the *lived experience* of students visiting the MSC, allowed analysis of the day-to-day work of tutoring in the centre and thus presents some novel findings. It is important to note that the results obtained are based solely on the *mathematical difficulties* exhibited by students who chose to come to the centre unlike diagnostic testing which examines those of the whole cohort. However, as a lecturer highlighted, in interviews described earlier, 'if a student is sort of working hard and going to the maths support centre … it gives me a much better feedback on the things that need to be focussed on."

The data were classified under 31 codes of *mathematical difficulties* and any tutor entry could be assigned to one or more of these codes. This allowed an in-depth analysis of the tutor entries. Lessons learnt from the pilot study and in particular the use of carbon copy to record the assistance given to the student by tutors working in the centre, allowed more rigorous reporting of the tutor entries.

Based on Table 3.5, the data shows that there are a number of codes with considerably higher levels of *mathematical difficulties*. These were *advanced*, *vectors*, *discrete mathematics*, *matrices*, *mechanics* and *continuous distributions*. These are discussed further below where they relate to Level 0 or 1 modules. Although *advanced* is among these, it is discussed separately in relation to the next research question.

A number of topics have been removed from the syllabi for the new Leaving Certificate mathematics, included among these are *vectors* and *matrices*. Some of the *mathematical difficulties* exhibited by students attending the MSC for these codes may relate to recent changes in the secondary education syllabi in Ireland.

Recent research (Prendergast, Faulkner, Breen, & Carr, 2017) indicates that lecturers are aware of the changes to the syllabi and a number of them have adjusted their courses accordingly.

However, this research has highlighted these codes as significant problem areas. This may suggest that the changes in the new Leaving Certificate mathematics syllabi have not been addressed sufficiently by lecturers in their module content and may therefore need further consideration.

Next we look at the *discrete mathematics* and *mechanics* codes. As discrete mathematics was not included in the Leaving Certificate syllabi and there are no changes to applied mathematics, which includes *mechanics*, these cannot be attributed to the revised syllabi.

The statistics syllabus has been increased in the new Leaving Certificate programme and as noted earlier some difficulties with *continuous distributions* were evident in Level 1 modules but many of the *mathematical difficulties* in relation to this code appeared in Level 2 as explained earlier.

In carrying out an examination of all codes it became clear on the analysis of the tutor entries that the data fell naturally into two different categories. One concerned basic *mathematical difficulties* and the other the module content. To analyse the data in this way required knowledge of the student's previous mathematical experience. This was available for students taking Level 0 or 1 modules as the vast majority of students enter UCD, with a Leaving Certificate in mathematics. The minimum mathematics entrance level is set for each programme, see Table 3.1. This level of mathematics, based on the syllabi for the Leaving Certificate examinations, was taken as the pre-requisite knowledge for each module. All tutor entries relating to Level 0 or 1 modules were analysed under 29 codes. The code of *partial differentiation* and that of *advanced* were not observed for Level 0 or 1 modules. Each *mathematical difficulty* was classified according to whether the difficulty arose as either *Prior Learning* or *Module Content* which were defined as follows:

i. *Prior Learning* — mathematical knowledge that is taken as pre-requisite knowledge for the given module,

ii. *Module Content* — mathematical knowledge taught in the module and not considered as pre-requisite knowledge.

The following are the codes where *mathematical difficulties* with *Prior Learning* were mainly evident:

- *Basic algebra,*
- *Indices,*
- *Graphs,* and
- *Trigonometry.*

The more interesting aspect of the findings in this study is where the *mathematical difficulties* required assistance with mathematical knowledge taught in the module and not considered as prerequisite knowledge. Four codes are particularly, notable in this respect. They are, in the descending order of number of difficulties:

- *Vectors,*
- *Discrete mathematics,*
- *Matrices, and*
- *Mechanics.*

These codes, classified almost entirely as *Module Content* represent the highest number of *mathematical difficulties* experienced by students attending the MSC over the eight-week period.

The classification of each *mathematical difficulty* as *Prior Learning* or *Module Content,* based on the module requirement and exhibited by the students attending the MSC, is displayed in Figure 4.2 where *Module Content* is shown in red and *Prior Learning* is in blue. It is clearly evident from this chart that *mathematical difficulties* exhibited in relation to *Module Content* is the major area of difficulty. The analysis demonstrated that problems arising relative to *Module Content* were more than twice as likely as those for *Prior Learning.* The further confirmation of this result was seen by eliminating results from students who attended on more than five occasions. This avoided large attendance by a student, skewing the data and thus strengthening the previous finding (see Figure 4.4). This is novel finding and was made possible by basing the research on the *lived experience* of students attending an MSC.

Evidence of students' *mathematical difficulties* in relation to *Prior Learning* had emerged through previous studies which focused on the analysis of diagnostic tests as seen in (Robinson & Croft, 2003; Faulkner, Hannigan, & Gill, 2010; Lawson, 2003; Ní Fhloinn, 2009a) or through examination of lecturers' and students' perceptions of these difficulties (Perkin, Pell & Croft, 2007; Ní Shé, Mac an Bhaird, Ní Fhloinn, & O'Shea, 2017). The outcomes revealed by these studies were similar in many cases to the results for *Prior Learning* in this study. For example*, mathematical difficulties* in *basic algebra* as discussed earlier was a concern and this was also reported by Jeffes et al., (2013) in an interim report, investigating the mathematical competencies of second-level students after the introduction of *Project Maths.* Here the authors noted that students performed least effectively in algebra and functions. Sheridan, (2013) stated that algebra and arithmetic were the two main areas of difficulty indicated by diagnostic testing. Similar results were reported in the chief examiners report (2015) when he remarked that many candidates displayed a lack of knowledge of standard procedures and a lack of basic competence in algebra and in algebraic manipulation. Prendergast, Faulkner, Breen, & Carr, (2017) asking lecturers to compare the mathematical performance of students educated in the traditional methods with those in *Project Maths* found that algebra was among the only strands indicated by the lecturers as worse or much worse. Prendergast and Faulkner (2018), also comparing diagnostic tests on incoming students before and after the introduction of *Project Maths,* focused on the strand *algebra*. The authors noted that the introduction of the new curriculum coincided with a decline in students' technical algebraic skills. Reports such as Treacy and Faulkner (2015) and Prendergast and Treacy (2015), examining the results of incoming university students' annual diagnostic tests, suggested that there was a decline in performance of the basic mathematical skills required for students studying in higher education and showed this decline was particularly significant after the implementation of *Project Maths.*

A recent study by Duggan, Cowan and Cantley (2018) conducted a series of semi-structured interviews with lecturers teaching first year mathematics across a variety of academic disciplines. The interviews with nine lecturers investigated their perceptions of new undergraduates' mathematical skills and the *Project Maths* curriculum. The authors observed several common findings regarding perceptions of *Project Maths* and the '*mathematical* *preparedness*' of new undergraduates. One of the main issues in the perception of the lecturers was that many new undergraduates lack some very basic concepts and skills, such as algebraic manipulation, fractions and the appropriate use of units. They stated that this finding was a particular concern for lecturers within the STEM disciplines. The feedback to lecturers from the *lived experience* of students attending the MSC showed that students had difficulty in the area of *basic algebra* and also, as discussed earlier with *indices* and limited difficulty recorded for *inequalities* among other basic skills*.*

However, the study using the *lived experience* of students attending the MSC provides a different approach in examining mathematical difficulties related to modules content. This was only previously seen in observations by lecturers' or students' perceptions of these difficulties. A number of papers examined differences in lecturers' and students' perceptions of the *mathematical difficulties* experienced by students. Although, many just considered basic mathematical difficulties such as Perkin, Pell, & Croft, (2007). A similar study to that of Perkin, Pell and Croft (2007), in the UK, was that of Ní Shé, Mac an Bhaird, Ní Fhloinn, and O'Shea, (2017). These authors conducted two surveys in Spring 2015. The first was a survey of students enrolled in first year undergraduate non-specialist mathematics modules in four HEIs. The survey aimed to identify the mathematical topics which students in these modules, determined as problematic and to detect if concepts or procedures caused the greater difficulty. The second survey was emailed to all lecturers teaching Level 1 undergraduate mathematics in Ireland. Thirty-two responses were received. The students were asked to rate their ability to answer forty-six mathematical questions and to answer seven open-ended questions. In the open-ended questions the students were asked to indicate if it was the ideas or methods that caused difficulty. The lecturers' questionnaire, had ten open-ended questions. The variation in location of the two surveys, one with students in the four HEIs involved in the study and the other with lecturers from HEIs excluding those involved in the study, may have limited us drawing conclusions from these comparisons. Furthermore, although the lecturers were all teaching first year modules it is not stated if these were the same modules as covered in the student survey and therefore the extent and topics covered may have varied. As a result, differences in terminology may have hindered a common analysis. Approximately two-thirds of the students surveyed had taken the Higher Level LC examination and one-third the Ordinary Level LC examination. The authors (2017) found that students who came to third-level having sat the Higher Level LC mathematics examination were more likely to mention integration as a problem whereas the Ordinary Level students stated they found logs difficult. Both of these areas, evident in the present study, were seen as areas of difficulty but not as the *mathematical difficulties* occurring among the highest frequencies. However, it is interesting to note that the level of integration covered in the present Higher Level Leaving Certificate syllabus has been reduced and logs are not covered in the Ordinary Level. This might explain the difficulties seen here particularly, if the lecturers had not been aware of the changes in the syllabi. In answers to the open-ended question in the student survey (2017) – '*What topics in first year caused you most difficulty? . . . Please indicate whether it was the methods or the ideas* *involved that made the topic difficult for you.' — the* students listed integration, differentiation, functions, logs and limits as difficult and rated their ability to understand higher than their ability to answer questions. What was especially interesting was that only a small number of students reported difficulty with matrices, vectors or algebra. All of these were shown to be significant in the *lived experience* study. However, as the authors (2017) commented most of the topics listed as easy by a number of the students, were listed by others as difficult and this is seen in answer to – '*What topics in first year did you find most easy?' — the students* listed algebra, equations and formula, differentiation and integration, functions and graphs, matrices, complex numbers, logs, statistics and vectors. So it is not easy to draw conclusions from the results of this report. In answer to the question — '*What procedures and tasks in the first-year curriculum cause most difficulty for your students*?' – the lecturers' responses mainly related to formulae, equations and symbols, fractions, linear algebra, logs and indices, differentiation, integration, functions and graphing, trigonometry, probability and statistics and geometry. The report (2017) stated that eight percent of students referred to finding topics easy or difficult depending on whether they had covered them before whereas lecturers found that students' difficulties with more advanced topics resulted from a lack of basic skills. The recent study by Duggan, Cowan and Cantley (2018) as mentioned previously found that all of the participating lecturers suggested that new undergraduates had difficulty applying mathematics in unfamiliar contexts. This might further explain the results of the *lived experience* finding that more than half of the *mathematical difficulties* exhibited by students related directly to *Module Content* as opposed to *Prior Knowledge.*

To answer the second part of the research question - how does this result compare to the original purpose of a Maths Support Centre? The Seminal report 'Measuring the mathematics problem' (Hawkes and Savage, 2000) noted a critical reduction in the mathematics ability of incoming students to third-level education. One of its recommendations that 'prompt and effective support should be available to students whose mathematical background is found wanting' (2000, p.iv), supported by the Smith Report (2004), legitimised the provision of mathematics support for students with weak mathematical backgrounds. Furthermore, diagnostic testing reinforced the need for student assistance with Prior Learning.

Although the original purpose of a mathematics support centre, as noted above, was to support students with difficulties with pre-requisite knowledge, more recently, mathematics support has been extended to cover a broader remit. Mathematics support, as noted by Croft, Lawson, Hawkes, Grove, Bowers & Petrie (2014), included providing facilities to 'support and enhance students' learning of mathematics or statistics whilst the student is enrolled on a programme of study'. This appears to suggest that assistance given may extend beyond prerequisite learning and the findings from the *lived experience* supports this notion, given the high proportion of module content observed. It is probable that *mathematical difficulties* in relation to *Module Content* has always existed but previous methods of exploring these has not furnished the data to allow the analysis in this way.

This study explored from the *lived experience* of students attending an MSC and employing a different approach, provided information not alone on the nature of students' weaknesses in relation to *Prior Knowledge,* as might be expected, but also where difficulties arose with the *Module Content*.

Next the third research question is discussed:

> *In what ways can knowledge gained from the data collection contribute to the efficient running of a Maths Support Centre? Specifically, how can the findings inform management's decision-making to ensure that all students who visit the Maths Support Centre can be appropriately supported in a timely manner?*

Responses to the above question includes discussion of efficiency of the MSC in: the use of *Hot Topics*, regulation of attendance, highlighting problem codes or modules, feedback from tutors and collaborative partnership with lecturers. The efficacy of the centre is discussed in relation to attendance from modules at higher Levels.

5.2 Use of Hot Topics for High Attenders at the MSC

Hot Topics are specialist topic workshops normally limited to between 10 and 15 students. They are designed and delivered by the MSC for a significant minority of students within a module whose learning may be held back due to some missing background or module prerequisite or where a small percentage of students are, for a variety of reasons, having difficulty in a particular module. They are run at a code/module level for several reasons; firstly, students are more likely to attend when the topic relates directly to their module content; secondly, there is a limit to the extent of coverage in any one *hot topic* and thirdly, extending material covered to additional modules may result in either inadequate or excessive coverage of material for individual students. The sessions are always organised with the module coordinator's consent and can be requested by the lecturer, students, MSC or a combination of any of these.

The introduction of more *Hot Topic* sessions could be one way of increasing efficiency. In Semester 1, 2014/2015 the MSC ran nine *hot topics*, five of which were held during the eight-week period of this research study. These covered the following topics: vectors, linear algebra, logs and exponentials, statistics and a basic mathematics session run for an introductory mathematics module.

Table 5.1 List of potential modules for Hot Topics

Module Name (Anonymised)	Code	Total
Business Maths	Indices	28
Business Maths	Differentiation	27
Business Maths	Logs	22
Business Maths	Graphs	21
Business Maths	Modelling	14
Business Maths	Functions	13
Linear Algebra for Science	Matrices	60
Linear Algebra for Science	Vectors	18
Calculus for Engineers	Integration	27
Calculus for Engineers	Differentiation	24
Calculus for Engineers	Basic Algebra	17
Calculus for Engineering	Functions	10
Number Theory	Discrete Mathematics	76
Applied Maths for Engineers	Vectors	44
Applied Maths for Engineers	Mechanics	17
Applied Maths for Engineers	Trigonometry	12
Statistics for Business studies	Statistics	29
Statistics for Business studies	Mathematical Expressions	20
Statistics for Business studies	Discrete Distributions	17
Applied Maths for Science	Mechanics	18
Applied Maths for Science	Graphs	14
Applied Maths for Science	Limits and Continuity	12
Applied Maths for Science	Critical Points	11
Linear Algebra for Maths Specialists	Matrices	31
Linear Algebra for Maths Specialists	Complex Numbers	18
Staistics for Science	Statistics	33
Staistics for Science	Discrete Distributions	10
Multivariable Calculus for Engineers	Partial differentiation	37
Physics for Engineers	Mechanics	20
Physics for Engineers	Vectors	12
Combinatorics	Discrete Mathematics	27
Applied Biostats	Statistics	25
Applied Mathematics & Mechanics	Vectors	19
Access Mathematics Module	Basic Algebra	13
Calculus of Several Variables	Partial differentiation	12
Foundations of Physics	Mechanics	12
Introduction to Mathematics	Indices	10
Mathematics for Ag	Basic Algebra	10
Mechanics	Mechanics	10

Four further *hot topic* sessions were organized after the completion of the research period in Semester 1. The areas of mathematics for these were: linear algebra, two for statistics, and further one for an applied mathematics module. Perhaps consideration given to organizing the latter four *Hot Topics* a few weeks earlier might have been more beneficial for these students. Each of the above nine sessions were organized for a unique module shown highlighted in the darker coloured entries in the Total column in Table 5.1 above. Two of these, logs and indices, were covered in a single *Hot Topic*.

It is clear from the above Table 5.1 that that other areas could be considered for further *Hot Topics.* For example, *differentiation* for the Business mathematics module, *differentiation* and *integration* for the Engineering module, or combinatorics for the discrete mathematics module might be a few for possible consideration. Very high attendance such as for Number theory are discussed in section 5.4 identifying module problem areas below.

The weekday attendance pattern over the eight-week period is shown in Figure 4.5 in the results chapter. Data looking at attendance by day of the week might have been helpful in identifying times where *hot topics* should be considered but local university factors need to be considered in conjunction with this. For example, Fridays had the lowest attendance levels of the week and appeared to be an appropriate day to run *hot topics* as there would be space available. However, the reason for smaller numbers on Friday is that a number of students return home to other parts of the country and often leave on Friday for the weekend and would therefore be unlikely to attend on that day. The data showed scheduling for *Hot Topics* was set after the MSC closed about 6pm and also earlier in the week. The probable reasons for this were that students had finished their lectures and were available at these times and the MSC was busy with one-on-one or very small group sessions during the day.

As stated earlier *Hot topics* would not be time efficient if targeted at modules where only a few students attended for a small number of *mathematical difficulties.* Table 4.14 in the previous chapter showed the breakdown of mathematical difficulties per module for the top quartile. It displayed for each code: the total number of mathematical difficulties in column 2, the total number of modules from which students exhibited *difficulties* in column 3, and in the final column the top quartile of these modules by attendance. As seen already this was to set to the absolute maximum numbers of modules possible to target for *Hot Topics*. It showed that this would be 20% of the total modules attending.

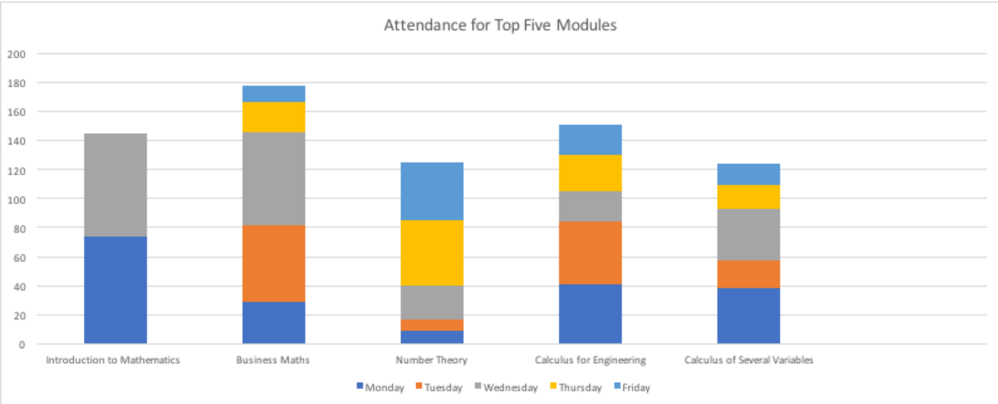
Visits per module occurring at less than 6 visits present a very clear picture of the extent of modules where the students from these modules show minimal attendance at the MSC. Where these are Level 1 or introductory modules, this may give an indication that there are other students that might benefit from attending the MSC. It may be, as a number of articles (Mac an Bhaird, Fitzmaurice, Ní Fhloinn, & O'Sullivan, 2013; Symonds, Lawson, & Robinson, 2008) suggest, that among the reasons for student non-attendance at mathematics learning and support centres were that they are unaware of their existence, location or were embarrassed to seek help. Direct contact with the module coordinator of these modules, and advertising the services of the MSC to a much wider audience, may be of benefit to these students.

5.3 Regulating attendance in the MSC

Overcrowding in the MSC may occur if students from a number of modules attend with high numbers on the same day. M*athematical difficulties* exhibited by students from some modules may be spread out evenly over the eight-week period of the study whereas for other modules *mathematical difficulties* may be seen with high numbers on a single day.

To see how this might cause a problem another aspect of attendance is explained below where five modules that have the highest number of *difficulties* were broken down by day of attendance.

Figure 5.1 Weekday attendance patterns at MSC for five modules with high attendance



The first column shows an introductory module where attendance is organised by the lecturer of the module. This lecturer was also a tutor in the MSC and arranged for any student in his class to come to the MSC on Mondays and Wednesdays at a given time and many of his students attended regularly. The second column is for the business Maths module. Quizzes, for which marks were given, were spread over Tuesdays, Wednesdays and Thursdays for this very large class of over 500 students. It can be clearly seen that attendance is highest on the days before the tests. The next column is a number theory module for which fortnightly workshops were due on Mondays and therefore attendance was high Thursday and Friday. The fourth column shows the attendance pattern for Calculus for Engineering and workshops were due at the weekend. However, the schedules for students studying Engineering is very tight and attendance at the MSC by these students was more likely to indicate free periods. The final column shows the Level 2 Multivariable Calculus for Engineers and once again they also have a busy schedule. The pattern of attendance, although only shown for a small number of modules when observed in tandem with Figure 4.5 attendance balances out for all modules over the week and additional crowding on any one day is not an issue.

Analysis of this data requires a working knowledge of the module specifics for example, weekly tests, workshops, mid-semester examinations among other data. Updated knowledge of these from the lecturer would be required for example, to alter the number of tutors attending on a given day. Therefore, communication, in this respect between lecturer and the MSC would be helpful in maintaining the efficiency of the MSC.

It is clear that this data would not be useful for an external analyst without in depth knowledge of the MSC and wide knowledge of module content and lecturers' practices.

5.4 Identifying module problem areas in relation to codes.

Table 4.16 displays the module which represented the highest number of *mathematical difficulties* for each individual code. The percentage of this number in relation to the total number of difficulties for the code is also shown. This might be useful in indicating problem modules in relation to specific codes. Where the numbers are high combined with high percentage of the total numbers for the code, contact with the lecturer over and above the normal feedback process, might be beneficial both for the lecturer and the efficiency of the MSC. Clearly there are a number of modules in the top half of this table that might be considered in this case.

5.5 Identifying module problems; intervention with lecturers for modules with high attendance relative to class size

Data may also be used to provide feedback to lecturers on individual modules disproportionately represented at the MSC by examining the data in a different manner. In Table 4.17 in the previous chapter each student attending the MSC relating to a module is counted only once for that module regardless of the number of visits they made. This is to counteract the skewing effect of multiple individual visits on the data. The table highlights modules that have a high number of students attending the MSC. This is additionally described with reference to class size. A small percentage of a class might be expected to attend the MSC and could imply individual student problems whereas a large percentage is more likely to indicate problems with module content. This information is therefore essential to allow feedback to lecturers in the case of serious concerns with certain modules. Some modules stood out in this analysis as seen in Table 4.17. For example, Applied Maths for Science with almost half the class attending the MSC for assistance with this module. Other modules that may also have needed consideration were Number Theory (42%) and Combinatorics and Number Theory (31%). The total number of visits by members of a module cohort is not known to the lecturer until the module has finished. Although in such cases as shown above, there is scope for the lecturer to make some adjustments "on the fly", it should be expected that there will be some significant change to the delivery of the module in the following year since a large proportion of students from the module seeking additional help is a strong indicator that all is not as it should be.

Three actions are key if the issues relate to difficulties with the content of the module. Firstly, it is important that lecturers read the feedback; secondly, that there is regular communication between the manager of the MSC and lecturers; and thirdly, that the MSC does not take on the position of replacing teaching or tutorials by MSC tutoring. This should remain the responsibility of the lecturer.

Lawson, Croft and Halpin (2003) considered this question further when the authors warned of the possibility of designers of programmes developing curricula not suitable for the level of students taking them and assuming the MSC would cover the mathematics requirement.

The present manager, in foreseeing these difficulties has initiated an MSC-Module Coordinator (MSC-MC) partnership agreement, renewed on an annual basis, both within, and outside, the UCD School of Mathematics and Statistics (see Appendix C for a copy of this agreement). Under this agreement, module coordinators and lecturers receive an automated email each Friday detailing the number of visits to the MSC from students in their module, the duration of each visit, the running count of the total visits over the semester and the nature of the student query and how it was remedied. In return, the module coordinators and the lecturers agree to engage with the MSC and in a situation where visits for a module are exceptionally high and regular, then, provision is made in the partnership on procedures in relation to ongoing MSC support for the module.

A further concern evident from Table 4.17 is the high percentage attendance of students studying Level 2 and Level 3 modules. We discuss this further in the next section, 5. Prioritization of access to the MSC. There we discuss a possible approach to increasing the efficacy of the MSC.

5.6 Prioritisation of access to the MSC

When Figure 4.6 and Table 4.17 in the previous chapter were combined they produced some important results. Firstly, we examined code/module attendance for Level 2. Figure 4.6 showed higher peaks in attendance for the codes, *continuous distributions*, *discrete mathematics*, *partial differentiation* and *vectors*. Table 4.17 identified particular modules showing concerns at module level. These were Statistics for Science, Mechanics, Vector Calculus and Linear Algebra modules. The syllabi for a number of these modules would relate to the coded difficulties revealed in Figure 4.6. For example: *continuous distributions* and Statistics for Science, *partial differentiation* and *vectors* for Vector Calculus.

Hot Topics were not normally considered for students in Level 2 except in the case of the two statistics modules where the syllabus for these modules was designed for students studying statistics for the first time at university and were therefore equivalent to a Level 1 module. Perhaps provision for further modules at Level 2 should be considered.

What is evident in Figure 4.6 was that for Level 3 there were very large numbers attending for one area, (code shown as *advanced)* but in this form it was not very useful as it did not break the code down into separate modules having difficulty. Table 4.17, on the other hand gave worthwhile information for Level 3 modules. This table showed that there were four Level 3 modules that raised concerns re attendance numbers at the MSC. This calls for a response from either the lecturer or the module co-ordinator. It could be that the previous mathematics modules are not preparing students appropriately for these courses and so there may need to be an element of course redesign. Alternatively, the method of delivery of the module may make it difficult for the students to comprehend the content and a solution would be to amend the way the content is delivered or it might be that the material is by nature hard and that extra support within the module delivery needs to be provided.

If efficiency of the MSC was the only consideration it might have been worth considering running a *Hot Topic* for Multivariable Calculus for Engineers or for Time series as seen in the table. However, efficacy of the MSC should also be considered. The MSC was aware from remarks by a number of students in Level 1 programmes that the centre was oversubscribed on many occasions. The manager had referred to the problems with overcrowding in the centre during the interviews with lecturers described previously in the results chapter. This he stated had discouraged some students from attending and others having to leave before they could be seen by a tutor. Therefore, a question that needed consideration was whether the MSC should cater for students at the higher levels when overcapacity problems needed to be addressed.

A decision was reached in the academic year 2015/2016, the year following this research, to restrict MSC support to students taking modules from Level 0-2 only. The reasons given for this change in operation of the MSC, were that the number attending for the higher levels was significant and visits from these students accounted for a disproportionate amount of tutor time (Cronin, 2016). This raises the question, however, of where these students in the higher levels can find help. It is evident from this study that they are seeking assistance either with advanced mathematical or statistical modules, or with projects requiring statistical analysis. There is therefore a blurred line here between MSC support and module support. Where the help is required in a module, lecturers generally have office hours, when any student may drop-in with an expectation that if their students have difficulty with any aspect of the module, they will come and talk to them. If statistical assistance is required by a student, CSTAR is a statistical service using one-to-one consultations, based in UCD and available to UCD students. The standard option is for an initial discussion of the student's needs and an estimate of costs, which is free of charge. Subsequent help must be paid for.

What is not clear, however, is why so many students beyond Level 1 sought assistance in the MSC rather than from their module lecturers. Croft and Grove (2006) suggested that problems are seen with specialist mathematics undergraduates in the years beyond transition to third-level. In particular, the issues they mentioned were in relation to disillusionment with mathematics and increasing drop-out rates. Croft, Grove and Bright (2008) had sought to identify factors which might have assisted specialist mathematics undergraduates to develop a more positive stance both in relation to their mathematical competence and their attitude to mathematics. What the authors proposed was extending the rationale of the MSC to the provision of a *'well-resourced social learning space'* (2008, p.12) for specialist mathematicians beyond the transition period to allow them to work together in groups and develop student learning communities. Solomon, Croft and Lawson (2010) also suggested that providing this learning space for mathematics undergraduates would allow them to create their own communities of practice and demonstrated that learning mathematics for a number of students is a social experience and many find it preferable to work in peer group situations.

5.7 Focus group held with MSC tutors

The main aim of the focus group was to enhance the efficiency of data input by gaining clarity and understanding of the MSC feedback process from the tutors' perspective and exploring with them means to improve the design of the procedure. The tutors had looked for clarification on a number of issues. These related to the time involved in data entry versus tutoring, the intended recipient of the data and the level of detail required. They made a number of proposals for more efficient data entry, such as improved drop-down menus specific to the module, availability of a free textbox and the ability to add a topic frequently requested by students. A difficulty noted by the tutors was where a student might wish to ask a very brief question in relation to a different module at the end of a session, this caused delay or incorrect module entry.

During the eight-week study, tutors had received regular critique of the feedback that they had submitted as part of the research project. In the following semester tutors continued to enter feedback data for lecturers and the MSC but did not receive feedback on the quality of their responses. In their discussion, tutors generally agreed that without feedback they felt they were operating in a vacuum, and that they found this challenging. It was clear from this that there is a need to close the information loop, in order to improve feedback and maintain motivation.

These improvements would undoubtedly decrease the time taken for the data entry process while still providing the pertinent information needed for both the MSC and lecturers.

5.8 Lecturers' feedback from the MSC

What feedback, if any, would be most beneficial for lecturers to receive on their students' visits to an MSC?

Mathematics support originated when concerned lecturers, recognising that many students were experiencing difficulty with the mathematics element of their studies, organised drop-in sessions once or twice a week for their students (Croft, 2008). As a consequence, these lecturers were fully aware of their students' mathematical issues. Many reports (Howson et al., 1995; Institute of Mathematics and its Applications, 1995; Sutherland & Pozzi, 1995b) in the UK focussed on the requirement to change second-level education. Universities, however, had to handle the reality of the consequential difficulties of their incoming students and the way they often dealt with this was through the provision of mathematics support. The establishment of mathematics support centres gradually gained momentum and the present extent of these centres in Ireland and the UK was seen in recent reports (Cronin, Cole, Clancy, Breen, & O'Sé, 2016; Perkin, Croft, & Lawson, 2013). In many cases, students are now assisted by mathematics support centre tutors, and lecturers may no longer have the same access, as previously, to the knowledge of *mathematical difficulties* that their students are experiencing. UCD MSC feedback is an attempt to close this gap and reconnect lecturers to this material.

The comparison by Kyle of the painstaking work and skill of a MSC tutor in analysing a student's *mathematical difficulty* with the challenge of locating the precise source of a drip from a leaking roof, was an apt analogy (Kyle, 2015). The UCD MSC believes that the analyses by tutors of students' *mathematical difficulties* are an invaluable source of information that can be recorded and provided as feedback to lecturers. However, it is essential, for this reason, to establish whether the lecturers find the feedback beneficial, and if so, in what ways do they find it useful. Responses by lecturers to this varied considerably. However, all lecturers confirmed that they found it worthwhile. Some lecturers suggested, in relation to the feedback, that it was more detailed and a better indicator of the difficulties that their students are experiencing, as compared to other forms of feedback. Others commented that it provided them with information of a more precise nature and importantly, it was directly related to issues that students were experiencing in their module. One lecturer remarked that what he found worthwhile was when a student asked for help in relation to material in a lecture or video and this was especially useful when a number of students attend the MSC with the same difficulty.

As Croft (2008) stated *'our primary concern must be the students'* (2008, p. 13). If a tutor is spending an inordinate amount of time recording data, they will inevitably have less time to tutor. Any recording in a MSC is time consuming, in particular any details that are added by tutors. Therefore, the question of what level of detail is required is relevant. The overall response of lecturers to this question recognised the importance of brevity, not only for the tutors, but also for themselves. Lecturers do not have time to read extensive feedback and although many lecturers were happy with the level of detail, others felt it was excessive. One interesting suggestion by a lecturer was that the increased level of detail might be useful at the beginning of a semester to indicate difficulties early on in the module. Provision of extra help at this time could be beneficial, perhaps in the form of *Hot Topics,* for those students who have issues with prior knowledge. Significantly the detail that a lecturer found most helpful was not only information in relation to the module topic but also the basic mathematical skill causing difficulty. They also found information relating to the number of visits, for the particular student query, useful.

The significance of the comprehensive coverage of their students' difficulties, as obtained in the MSC feedback, was seen by a number of lecturers as important. A lecturer wishing to make changes to their teaching approach might therefore use the MSC feedback to observe the effects of a change in method or approach to lecturing. It is possible as a comment made by one lecturer indicated, that even teaching the same module, the dynamics of a class may change from year to year and a lecturer might find the feedback of even greater value in future years. the fact that the information is collated by the MSC was particularly beneficial for future use, either to alter module content, or alert lecturers to possible problems in the following year. One lecturer commented that he made changes to his lecture material as a result of feedback he received from the MSC. While other lecturers saw the feedback as formative compared to other types of feedback they received through module tutoring or analysis of examination performance. Potential benefits of MSC contemporaneous feedback could be real time adjustment of lecturing approach, style and content.

Reports such as Treacy and Faulkner (2015) and Prendergast and Treacy (2015), examining the results of incoming university students' annual diagnostic tests, suggested there was a decline in performance of the basic mathematical skills required for students studying in higher education and showed this decline was particularly significant after the implementation of *Project Maths*. A study by Ní Shé, Mac an

Bhaird, Ní Fhloinn and O'Shea (2017) examined responses by lecturers and students to two surveys the purpose of which was to identify mathematical topics that are problematic for Level 1 students. The authors stated that the lecturers were most concerned with students' lack of ability in some basic algebra and concluded 'lecturers found that many of the problems students have with more advanced topics are related to a lack of these basic skills' (2017, p.719). The study by Duggan, Cowan and Cantley (2018) observed several common findings regarding lecturers' perceptions of *Project Maths* and the '*mathematical preparedness*' of new undergraduates. One of the main issues in the perception of the lecturers was that many new undergraduates lack some very basic concepts and skills, such as algebraic manipulation, fractions and the appropriate use of units. The feedback to lecturers from the *lived experience* of students attending the MSC showed that students had difficulty in these areas but more than half of the *mathematical difficulties* exhibited by students related directly to *Module Content* as opposed to *Prior Knowledge.* Although, the feedback is limited to those who attend the MSC it is beneficial to lecturers in that in provides feedback not alone on pre-requisite knowledge but also on the module content.

Different approaches to learning, teaching and assessment have been developed with the introduction of the new mathematics approach to post-primary mathematics initiated in *Project Maths* and finally implemented in full in 2017. A number of lecturers mentioned that the difficulties recorded in the feedback were familiar to them while other lecturers commented that they were surprised, on occasion, by certain difficulties that they had not previously observed. An increase in difficulty with *vectors* was mentioned by one lecturer, while reading his module feedback, than that which had been previously observed by him. Among the changes in the new Leaving Certificate syllabus, were: an increase in the proportion of the syllabus dealing with statistics and probability, the removal in totality of *vectors* and *matrices*, and changes (mainly reductions) to the material on functions and calculus (SEC, 2015). Lecturers may be informed of the new curriculum, yet the exact implications for first- year students may not be fully clear for a number of years. A recent study (Prendergast, Faulkner, Breen, & Carr, 2016), presented at the European Conference on Educational Research (ECER) analyses responses, by 44 lecturers teaching in Irish third-level universities and Institutes of Technology to a questionnaire aiming to investigate how third-level mathematics lecturers perceived the new mathematics for the Leaving Certificate and enquiring whether they had made any changes to their teaching with the introduction of *Project Maths*. The findings of the study showed that although the lecturers were aware of *Project Maths* 'they are not aware of the changes in full and how it may impact upon their own course content, teaching and assessment strategies' (p.2). Perhaps, the learning outcomes on the new syllabi are not clear enough or sufficiently detailed to allow lecturers to understand fully the extent of coverage of some mathematical areas on the new Leaving Certificate syllabi and feedback of *mathematical difficulties* exhibited by the *lived experience* of students attending the MSC may be beneficial in this respect.

Having established the benefits of lecturer feedback it is now necessary to look at what feedback is most beneficial. To do this the opinions obtained from the lecturers' interviews are considered. A common theme expressed by the lecturers was the necessity for brevity. Lecturers do not have the time to read in-depth feedback from individual students and would instead favour a brief overview of the problem area. This they suggested should include both the basic skill and any difficulty with the lecture material. One approach suggested was that a drop-down menu of topics covered in the module and supplied by each lecturer, would be advantageous to both lecturer and MSC. The number of students seeking assistance for a particular issue was also an important consideration for the lecturers. A lecturer would be most concerned if high numbers of their students attend the MSC for difficulty relating to their module. As discussed easrlier where there are high levels of attendance for a specific area, *Hot Topics* should be run. The value of these sessions was confirmed by a number of lecturers and it was further suggested that feedback on the content of Hot Topics would be useful. Lecturers noted that benefits of the feedback were considered to be most useful to lecturers teaching modules for the first time.

Based on the data there may be an increase in difficulties with module content arising from changes in Second Level syllabi. To address this, additional feedback to lecturers following syllabi changes may be worthwhile.

Although the data in this research were only based on those students attending the MSC, when lecturers change their lecturing practice as a result of feedback from the MSC it may also prove beneficial for students who find the same difficulty but for some reason do not choose to come to the MSC.

Chapter 6 Conclusions

This study employed the *lived experience* of students attending an MSC over an eight-week period to explore students' mathematical difficulties exhibited at third-level. The research furnishes information not alone on the nature of students' mathematical weaknesses in relation to *Prior Knowledge,* as previously examined in many reports, but also demonstrates where difficulties arose with the *Module Content*. In this respect the research provides an original contribution to the existing body of work in this field.

The seminal report 'Measuring the mathematics problem' (Hawkes and Savage, 2000) noted a critical reduction in the mathematics ability of incoming students to third-level education in the UK. One of its recommendations that 'prompt and effective support should be available to students whose mathematical background is found wanting' (2000, p.iv), legitimised the provision of mathematics support for students with weak mathematical backgrounds. Mathematics support centres were mainly introduced as one option for providing this support. The UCD MSC was set up in 2004 with the original motivation, like many other centres, to assist students in the transition from post-primary to university education. The focus was to support in particular, those students at Level 1 who might experience difficulty in their mathematical modules arising from weakness in their prerequisite knowledge. Over time, this expanded to provide assistance to all levels and programmes within the University.

Previous studies measured the extent of students' *mathematical difficulties* by analysing results of diagnostic testing, or they examined these difficulties via surveys of lecturers or students. Diagnostic tests can be beneficial in revealing deficits in the prior knowledge or basic skills of the incoming cohort and allowing the provision of extra support where it is needed. They are also useful in illustrating overall yearly trends in the mathematical competencies of incoming students. In contrast, the approach taken in this study, unlike previous work done in this area, was based on the *lived experience* of students attending a mathematics support centre over an eight-week period. It therefore adds to the overall knowledge of the *mathematical difficulties* faced by students in their transition to third-level education. It aligns more specifically with the third-level content; although, it is important to state, it is limited to those students who seek help in the MSC.

In carrying out an analysis of the tutor entries it became clear that the data fell naturally into two distinct categories. One concerned basic *mathematical difficulties* (prerequisite knowledge) and the other related to difficulties with module content.

The results from the *lived experience* with respect to prior knowledge were broadly in line with other reports such as those from Carr, Murphy, Bowe, & Ní Fhloinn & O'Shea, (2013) and Ní Shé, Mac an Bhaird, Ní Fhloinn, & 0'Shea (2017) among others. The most frequently occurring prerequisite area of difficulty for the students was algebraic manipulation. Issues were also evident with factorisation, indices, algebraic fractions and logarithms. Sketching linear and quadratic functions was seen as a difficulty in calculus and in applied mathematics *trigonometry* caused problems. The major area of difficulty in statistics was understanding normal and t-distributions and reading their respective tables.

However, exploring the data it was clear that assistance was given to students on difficulties related not simply to prior knowledge but also to content taught in the module. Moreover, help given to students for difficulties relating to *module content* were seen to be almost twice as frequent as those for *prior knowledge*. This finding is novel.

Assistance with module content was shown to be particularly high for a number of mathematical areas such as *discrete mathematics, vectors*, *matrices* and *mechanics*. Some of these may have arisen partially as a result of changes in the mathematical syllabi for second-level education in Ireland. Namely, *vectors* and *matrices* are no longer covered on the revised Leaving Certificate mathematics examinations. Given that the data in this study was collected in the first year in which these changes were implemented, this may indicate that module content has not been adapted sufficiently to the prerequisite knowledge of incoming students. The changes to the Leaving Certificate did not impact Mechanics, but unlike the A-levels syllabi in UK, this is not included in the Leaving Certificate mathematics syllabi. It is available as part of a separate subject (Applied Mathematics), which only 4% of those taking the Leaving Certificate examinations studied. Discrete mathematics had been discontinued for these examinations many years previously.

An important question arising with this finding on module content is how this compares to the original purpose of a mathematics support centre? Many lecturers rightly feel it is perfectly fine for a small number of students to go to the centre for assistance with prerequisite knowledge required for their module but that students should be supported on the actual content of the module by the lecturer and his tutors. This should certainly be true for the majority of classes. However, with expanding access to third-level education, resulting large classes and widening diversity in prior educational achievement some students may need considerably more assistance to succeed at third-level. It is not just accessing third-level education but continuing successfully to completion that may be a problem for these students. There may be a myriad of reasons for this. Perhaps with increasing equity in education, their level of mathematics on entry is not high enough, they lack confidence in their mathematical ability or come from disadvantaged backgrounds. The one-on-one help available at the mathematics support centre can be invaluable to their ultimate success. If as suggested by Lawson (2012, p.4) 'a primary feature of mathematics support is helping students to achieve their full potential – whether they are struggling to pass or have realistic hopes of gaining a first', then the reality of the assistance given in a support centre has necessarily changed as seen in this study.

A small percentage of a class might be expected to attend the MSC and could imply individual student problems whereas a large percentage is more likely to indicate problems with module content. Table 4.17 highlighted a number of modules that had a high number of students attending the MSC with reference to class size. Evidence of this has not been previously published and is a further original finding arising from this study.

Although the remit of mathematics support centres has expanded beyond their original purpose, priority should still be Level 1. The research showed that over thirteen percent of attendance at the MSC related to students from Level 3 and 4 modules. There may be valid reasons for this but when the number of visits to the centre became almost unmanageable in 2015/16, data collected underpinned the decision to restrict support to students from Level 0, 1 and 2 modules.

Dangers exist for support centres where a high proportion of students attend for assistance with module content. A possible solution in these cases might be the introduction of the 'Maths Support Centre - Module Coordinator Partnership Agreement' as discussed earlier. An on-going line of communication between the lecturer and centre is important, including the significance of the provision of detailed information to and from the lecturer on a regular basis.

The anonymised real-time feedback of tutor entries to lecturers as seen in this study is one method of securing this. All lecturers interviewed agreed they found this useful but stated that brevity of content was essential. Some important aspects of the feedback were suggested by the lecturers. Among these were – the numbers of their students seeking help, the topic of prerequisite mathematics needed and the area of module content covered. Integral to this, is the quality of tutor input to the system on each student visit. Tutors suggestions may benefit the efficiency of the centre by reducing time spent entering the data. Tutors also emphasised the importance of two-way communication to improve the content.

Where students studying a module needed assistance with basic *mathematical difficulties* that were a prerequisite for the module or were having difficulty with the module content the lecturers supported the utilisation of Hot Topics. However, it was agreed that these should only take place with agreement between the module lecturer and the support centre and where the percentage of the class needing this support was low. This research showed that although five Hot Topics had been organised in the eight-week period, holding Hot Topics for further modules might have benefitted the efficiency of the support centre by reducing the attendance of students for one-on-one assistance.

It is probable that students' *mathematical difficulties* in relation to module content has always existed but previous methods of exploring this had not supplied the data to allow the analysis in this way. The research suggests there may exist more of an overlap between the support provided by the lecturer and tutor of a module and that given in a mathematics support centre, than previously observed.

Further Research

Topics for further research might include an examination of the ways in which the receipt of feedback each week impacts a lecturer's practice, if at all. In addition, it may be of interest to examine the impact of this feedback on a novice lecturer's practice. The MSC manager, Dr Anthony Cronin, is interested in exploring these issues.

The new mathematics syllabus is now well established. It would be interesting to investigate if with the constraint on time available to teach and the critical importance of the Leaving Certificate examination as the gateway to third-level education, second-level teachers have changed their approach and practices of teaching in the new mathematics programme or do they revert to old ways when preparing their students for examinations.

Recommendations

- *Accurate and reliable electronic data systems linked to the university records data are very valuable for MSCs and institutions should make resources available for the provision of such systems.*

- *The role of the lecturer and the MSC should be clearly defined. A contract agreed between MSC and lecturer to enable the maintenance of two-way communication would be useful.*

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Appendix A

Stage 1: Initial databases (2009-2013)

The UCD MSC database was initially set up on a MySQL server in 2009. For each academic year, the database contained a table named *attendance* containing a number of fields (or columns). Among these columns, as stated previously, are two which were populated by the tutors in the MSC. The first is the *mathematical topics* column. This column contains a summary of the assistance given to the student and this was added to the database by the tutor after working with an individual student. The second column was to allow tutors to enter *basic mathematical difficulties* experienced by the student, if applicable. These columns will be referred to respectively, as *mathematical topic* and *basic difficulty* entries*,* when discussing the data collection up to 2013.

Structured Query Language (SQL) is used to retrieve the contents or part contents of a field (column) for analytics purposes. Using SQL, data or subsets of data could be retrieved from both columns described above. Table A.1 gives some information downloaded from these fields. Firstly, the total number of entries in the *mathematical topic* column*,* including the blank entries*,* in each of the four years is given. The number of entries where the *mathematical topic* entry was blank is shown in the second row. The bottom row counts the total number of entries recorded in the *basic difficulty* column*.*

The low number of blank entries in 2009-2010 and 2010-2011 can be explained by the fact that tutors, in these years, kept a hard-copy of student ID and a very brief description of the topic covered. If the tutor had no time to fill this in on the database, it was added by the manager at a later date. This use of hard-copy was discontinued in the later years as it was felt that it added unnecessary workload.

Table A.1 Summary of data count for Stage 1 of research

Entries	2009-2010	2010-2011	2011-2012	2012-2013
Number of *mathematical topic* entries	3508	4293	4401	4750
Number of blank entries	80	19	491	482
Number of *basic difficulty* entries	7	8	51	4

In UCD MSC students are encouraged to work in groups where possible, especially when they arrive together with similar questions. A number of blank entries, in the later years, may have arisen as a result of students arriving in small study groups to work on their own. These students logged onto the system but may not have sought assistance from a tutor. Also the annual numbers attending the MSC and the average length of time spent by a student in the MSC increased in 2011-2012 and again in 2012-2013, as seen in the Annual Reports (www.ucd.ie/msc). This would indicate that the MSC was busier in these years and tutors may not have had time to enter data. The blank entries are frequently observed during the middle of the semester when many lecturers schedule midterm tests and again at the end of semester, prior to the final examinations. At these times the MSC is at its busiest.

Mathematical topic and *basic difficulty* entries could not be added to the database at the same time. *Mathematical topic* entries were always given priority and time was rarely available for entering a *basic difficulty* and this is the most likely explanation for the low number of entries in this category. Analysis of the 70 entries of *basic* *difficulty,* although sparse in nature, provided limited evidence of codes that could represent basic mathematical areas. For example, algebraic difficulties were indicated by the following entries:

- *Basic algebra*
- *a/b/c = (a/b)/c*
- *Cancelling error (x+h)/h = x and*
- *Factorisation*

Many *mathematical topic* entries had no example other than stating the difficulty, as seen below:

- *Normal sub-groups*
- *Statistics*
- *Revision and*
- *BOMDAS*

With little information revealed on the *mathematical difficulties* experienced by students attending the MSC, either in the previous four years' *mathematical topic* entries or in the *basic difficulty* entered separately, a new approach was needed. From what was available in the data and from experience of working in the MSC, nine experimental preliminary codes that represented some of these difficulties, were chosen on a trial basis. The next step was to search the original data for these codes because it was felt that narrowing the data down, in this way, might provide more specific information on each proposed code. SQL is useful in searching for specific data in a database. It displays any word or part of a word sought by the query.

In Table A.2, for example, is the number of entries for each code in each year, obtained by searching the database under a word search as further described below.

Table A.2 List of codes and the number of each code annually from 2009-13

Code	2009-10	2010-11	2011-12	2012-13	Total
Algebra	107	192	110	69	478
Probability	48	88	122	136	394
Statistics	28	134	34	22	218
Indices	7	42	90	70	209
Simultaneous equations	2	31	40	92	165
Logs	26	40	25	34	125
Inequalities	2	6	33	13	54
Converting units	4	3	18	9	34
Factorisation	7	1	6	12	26

With even a cursory glance at the entries under any given code above, it showed that the majority either gave no information or else a short surface description of the problem. For example, from searching for *indices*, a large number of entries simply stated *indices* with no further explanation. Many entries under a search for the word *log* simply stated *problem with logs* or simply *logs* although a very small number did give some detail such as seen in the following entry:

log(1) = 0; ln(x^2) = 2 ln(x).

Past experience in the MSC made the researcher aware that students experience significant difficulties in these areas but the inputted entries were not capturing the detail of these difficulties.

There were also issues with using word searches. The first was that using word searches to identify codes may either produce extraneous items or miss legitimate entries. For example, when searching the word *log*, everything that had *log* within the word, was shown. This literally meant that not only entries such as shown above, but also *psychology, methodology* and other data with no relation to logarithm were displayed. Examining entries under the other codes as shown above provided similar findings. For example, some of the entries that were classified under *factorisation* and searched for under *factor* were as follows:

- *factorisation, cancelling;*
- *factorising and critical points;*
- *factorising and simplifying expressions;*
- *factorising;*
- *factorising cubic equations;*
- *factorising of limits, factorisation of quotient functions;*
- *factorisations of sums of polynomials;*
- *difference of two squares, and showing sum of two squares cannot be factorised;*
- *solving simultaneous equations, factorising quadratic equations; and*
- *removing a common factor.*

Searching for a word, using SQL as explained above, gave all entries that contained the word *factor* or those where *factor* is part of another word. But it did not bring up for example, *extracting 'a' from '(a^2 x + a)'.* It therefore did not find any factorisation in the database that was written only in the form of an algebraic expression, so these entries would not be picked up. It was therefore difficult to ensure the full extent of correct entries rightfully coded under *factorisation*. There would probably be some form of a factorisation not caught in the search and this was similarly true for all codes. Of course there was the possibility of coding the data at the end of some process of collecting more detailed data but the question then arises whether this would be the best approach? It was felt, that working with the tutors and explaining what data should be collected and for what reason the data was needed, was an important element in obtaining the detailed data required and that this would be more successful if the tutors were also involved in the coding process. Furthermore, it was realised that even if the entries contained sufficient detail to be useful, the data retrieval and analysis step of the study could be greatly enhanced if the entries could be coded by the tutor when entering them in the database in such a way that would allow a search for them using an SQL query. This is discussed further in Section 3.5.2 below.

The limitations of the data previously collected were apparent and the focus now became the collection and coding of more detailed data while bearing in mind that this had to be recorded, in a timely manner, by tutors working in a very busy MSC. In observing the paucity of both the number of *basic difficulties* entered and the lack of detail in the *mathematical topic* entries, the realisation of the importance of requiring tutors to input a single entry rather than two separate entries but more importantly, the need to train tutors to record detailed accounts of the mathematical issues with which students experience difficulty, became the priority. Firstly, the challenge was to determine what level of detail was required in the data entries and secondly, to work with the MSC tutors to ensure they understood the nature of the data required. Thirdly, ways had to be found such that high quality entries could be recorded and coded as accurately and efficiently as possible in a high-pressure busy MSC. The development of this data collection process is described in the following sections.

Appendix B

Stage 2: Refining codes and working with tutors (Semester 1, 2013-2014)

To address the research questions in Section 3.4, it was realised, as stated previously, that it was necessary to develop a process to enable tutors in the MSC to record reliable and detailed data on students' visits in a timely and efficient manner. In October 2013, Dr Anthony Cronin had just taken over as MSC manager. He gave his full support to the research proposal and agreed to meetings with the MSC tutors.

Tutors in the UCD MSC are mainly PhD students and frequently spend a number of consecutive years working in the centre. The tutors are generally excellent, well-experienced and enthusiastic teachers. The majority of tutors, at this time, had previously worked together in the centre.

During the first semester of 2013-2014, with a view to extracting codes using SQL, and realising the difficulties involved when trying to pull out the codes shown in Table 3.4 as described earlier, the idea of attaching a specific key to each code developed. Experimenting with different methods, eventually it was found that any key, entered within curly brackets, was easily extracted.
Evidence for these initial experimental codes was subsequently sought from the data collected in the first few weeks of semester 1 2013-2014. Examples, multiple in many cases, of each of the codes other than the code *sign rules*, were found and a few of these are shown below:

- *statistics problems, normal distributions and mean/standard deviation from given data {stat};*
- *student could not convert mg/mL to g/L or any other conversion. Student knew 1000mg in a g and 1000mL in a litre but did not know how to convert {cu};*
- *factoring quadratics, -b formula, roots of quadratics, graphing quadratics {f}; and*
- *plotting, identifying and estimating logarithmic functions {l}.*

Table B.1 Initial codes and respective keys

Code	Key
Basic algebra	{a}
Basic Statistics	{stat}
Basic Probability	{p}
Converting units	{cu}
Differentiation	{d}
Factorisation	{f}
Indices	{i}
Inequalities	{in}
Logs	{l}
Sign Rules (+/-)	(s}
Simultaneous equations	{se}

Factorisation is an obvious element of the code *algebra* but because of its frequent appearance it was maintained as a separate code. What remained coded as *algebra* was then recoded as *basic algebra. Statistics* was recoded as *Basic Statistics* and *probability* was recoded as *Basic Probability* for purposes of clarity. *Differentiation* although not sought in the initial search was frequently located in the data subsequently. It was decided that the difficulty with *sign rules (+/-)* was likely to have occurred but was not apparent, perhaps, due to the lack of detail in the data at this time. Therefore, it was decided to create a new code for this. The codes, with minimal alteration in the code name already presented in Table 3.4 are now shown with their respective keys in Table B.1 above with the addition of *sign rules* and *differentiation*.

The intention was that the tutors would simultaneously record and code, using the respective key, the *mathematical difficulties* experienced by their students. To achieve this, the tutors were required to:

- *Record each of their tutoring sessions in sufficient detail, to explain any basic mathematical difficulty that was preventing the student moving forward, and*
- *To simultaneously code the data, where relevant, by adding the appropriate key or keys for each tutor entry.*

What tutors were being asked to do, in real time, was to carry out a primary coding of their entries in such a way that subsequently, each coded area could be extracted by its key for further examination and determination. The commitment and ability of tutors, to record and simultaneously code high quality data, was an essential element in the research process and therefore the next step involved a meeting with tutors to communicate and secure their acceptance of the proposed method for data coding and collection. These meetings were informal at this stage of the research process.

In the third week of October 2013, the tutors were emailed to explain the proposed process of collecting and coding of the data. Attached were the contents of Table B.1 above with extra codes for which there was strong evidence found in the *tutor entries* of the previous weeks. These related to *trigonometry* {t}, *vectors* {v}, *sets* {sets} and *graph*s {g}. Also, included in the email, were some examples of detailed data coded by the addition of the respective key in each case. The following are some of the examples:

- *basic algebra x.x^2 students believed this = x^2 even though they knew ab meant a x b so add {a};*
- *x/2 one student did not know this = ½ x so add {a} and another student gave it = x^-2 so both index and algebra problem so add {i},{a}. NB separate codes with comma;*
- *trigonometry example, student did not understand where Cos, Sin and Tan had different signs as you went from 0 to 2pi. So put {t}.*

The next month was spent training the tutors, speaking and working with them in the MSC on a daily basis and/or emailing them regularly to explain precisely the method of entry and the quality of the data needed for the research. During the last few weeks of the semester this was not practical due to the numbers attending the MSC for help. However, the process of contacting tutors for further explanations of their entry by email, as had been taking place during this time, was continued. An example of this type of communication with the tutor is described below. Each entry on the database was available to view and copy in the following form.

Student Number	First Name	Second Name	Programme	Module Covered	Tutor Name	*Tutor Entry*	Date and time-in

When it was necessary to query a *tutor entry* with a tutor, a copy of their respective entry was emailed to the tutor with a request for extra information. For example, the following email was sent to a tutor who had not entered any keys in their *tutor entry*.

'I have added another code covering reading data from a graph {g} to the list. So am I correct if I add {g}, {d} to this entry. . . ?'

It included the view of the tutor's entry on the database. as shown below. Note the *tutor entry* is shown as is the date and time-in but other data have been removed here for the sake of anonymity.

Student Number	First Name	Second Name	DN250	Module covered	Tutor Name	*Interpreting differentiation* *Graph Reading*	2013-11-14 11:58:54

This is how the tutor replied:

'{g} should be added. The student was able to find the derivative of a function but not deduce information about the function (that f'(x) < 0 means the function is decreasing, for example). I wasn't sure if the {d} tag should be added but it makes sense if it is.'

In this case, additions would have been added to the *tutor entry* column and the final entry would have read as follows:

'Interpreting differentiation, Graph reading, the student was able to find the derivative of a function but not deduce information about the function (that f'(x) < 0 means the function is decreasing, for example) {g},{d}.'

This entry, as seen above, would then appear if a search, using the respective code key, was made for either coding, that is either *graphs* or *differentiation*.

Here is another example. A tutor was emailed the following extract and asked:

'Was this long division in algebra, the factor theorem or what exactly was the problem? Perhaps I should add {a}? '

Student Number	First Name	Second Name	DN250	Module Covered	Tutor Name	*Factoring cubic equation and polynomial division*	2013-11-14 10:04:27

This is how the tutor replied:

> *'This was the factor theorem. The student was trying to factor cubic equations and knew how to find the first root/factor, but not what to do then. So I showed her how to use polynomial long division to find the remaining quadratic. You could add {a} here.'*

So this was the final entry:

> *'Factoring cubic equations and polynomial division, the tutor said that this was the factor theorem. The student was trying to factor cubic equations and knew how to find the first root/factor, but not what to do then. So I showed her how to use polynomial long division to find the remaining quadratic. {a}'*

At this time, training of the tutors, in accurate coding and detailed data entry, was the main concern. In most cases these changes were made by the researcher. But as tutors became accustomed to the data entry process, queries reduced and many tutors adjusted their own entries to add the extra coding and/or detailed data. However, the responsibility to check the entries always remained with the researcher.

It was while working with the tutors that the requirement for extra codes arose. Tutors were encouraged to contact the researcher if, in their opinion, additional coding of areas of *mathematical difficulty* would be appropriate. The MSC tutors, through understanding the recording and coding process in greater depth, suggested new codes that they felt should be included. For example, two tutors emailed with suggestions for further codes including *matrices* {m}, and *limits* {l}.

Table B.2 Extra codes and respective keys added

Code	Key
Critical points	{cp}
Fractions	{fr}
Functions	{fun}
Graphs	{g}
Limits and continuity	{lim}
Mathematical expressions	{mexp}
Matrices	{m}
Pattern spotting	{pspot}
Sets	{sets}
Trigonometry	{t}
Unit Vector	{uv}
Vector	{v}
Word problems	{wp}

Extra codes, with respective keys, were added as shown in Table B.2 above. Data for the research, as stated, needed careful coding and a detailed explanation of the difficulty encountered by each student. Working with tutors to ensure understanding of the proposed process of entering data and to provide clarity on the contents of each code, was continued over the next few weeks. It was realised by adding the extra detail that this would entail considerable work for the tutors but would be of value, eventually, in the final analysis of the data. Updated copies of the codes and respective keys were made available, in the MSC, in laminated form.

A meeting was held in mid-January 2014 with eight experienced MSC tutors, to inform them of the pilot study which was planned for semester 2, 2013-14 and to present them with the new list of twenty-three codes with their respective keys. These were as shown as the combination of Tables B.1 and B.2 with the codes *unit vector* and *vector* combined under *vector* as the only alteration implemented. To further clarify with these tutors, the nature and quality of the *tutor entries* that should be collected, the following two examples were used to describe the difference between a valuable and a less valuable *tutor entry*:

> ***Example A**: A student had a problem with limits and continuity and also a problem factoring out 'h' and expanding in a question on first principles {a}, {s} {lim};*
>
> ***Example B**: A problem simplifying an expression – common denominator {a};*

where {a} represented an algebraic difficulty and {s} a problem with plus or minus signs. It was explained to the tutors that it is unclear in Example A where the student's difficulty lies. Is it a question of expanding the square or cubic brackets? What is the problem with *limits and continuity*? In Example B the student's difficulty is stated much more clearly. The student is unable to simplify the expression using a common denominator.

Suggestions were also sought from the tutors at this meeting on how the efficiency of the data collection might be improved. As a result of the meeting and further discussions, the tutors provided very helpful suggestions. Among these were, the introduction of further codes, the use of 'pseudo-LaTex' for entering data and the innovative idea of using carbon-copy notebooks.

Appendix C

Group of Mathematical Difficulties: Number of Mathematical Difficulties less than 30

Grouping 1: Algebra

Fractions

All students seeking help in this mathematical area were from Level 0 or Level 1 modules. Included in this code are very basic difficulties with numeric and algebraic fractions and the recognition of the use of reduction in fractions to help solve more difficult questions. The visits coded under *Fractions* could be described as follows:

Basic understanding of fractions

The contrast in the type of assistance needed in different modules is seen in *tutor entries*. A very basic understanding was required by a student in an Introductory Mathematics module as seen here:

> *'How to show more complicated numbers on a number line, how to write a number like 6 5/19 as a decimal number to show it on a number line.'*

A more detailed understanding of *fractions* was needed for example with eight students from Category B Calculus module as seen by the following example:

> *'There was a problem with fractions though and went through an example on how to add multiply and divide fractions,*
> $\frac{1}{2}+\frac{1}{3}-1$ calculated by taking a common denominator $1 \div \frac{1}{2} = \frac{1}{1} \times \frac{2}{1} = \frac{2}{1} = 2$.'

Understanding algebraic fractions

Once again the contrast in assistance required is shown in the following examples. A student studying a Category A Calculus module had the following basic misunderstanding:

> '*[Student] wanted to know if you could cancel the* x's in $\frac{x}{x+y}$ or separate out $\frac{1}{x+y}$.'

Factorisation

This code included student difficulties with factorising mathematical expressions and solving equations, mainly recognising common factors in complicated expressions and solving quadratic equations. Students also demonstrated that they had problems solving cubic equations. Students' difficulties can be described as follows:

Taking out a common factor

Difficulties here, in certain *mathematical difficulties*, related to very basic expressions as seen below. This *mathematical difficulty* was experienced by a student in a Category A module:

> *'How to factorise e.g. 4x + 16y. Student really didn't understand the concept and why you would just 'take out' 4. (Tutor) went through it step by step.'*

But these difficulties were also evident when students studying a Level 2 *Calculus* module for non-mathematics majors, attended the MSC with the following query:

> *'Students were finding the value for E in an electric field which was sum of E_1+ E_2 + E_3 + E4 each of the E values were very complicated but each was multiplied by the constant, calling this constant k and take it out from each E and so expression was much neater.'*

Recognising and solving quadratic or cubic equations

Examples here include factorising simple quadratic equations and solving cubic equations. Common problems were using the '-b formula' or recognising the solution to a quadratic in the form $ax^2 + bx = 0$ particularly when it was given with a variable other than *x.* A number of students from a core module for Mathematics and Physics undergraduates found difficulty solving the following quadratic equation:

> *'How to find the fixed points of $Solve\ u^2 + u(A-1) = 0$. Student was confused I think by how complicated it looked, once I pointed out that in ax^2+bx=0, there was no c, student realised that they could take out the u.'*

Omission of certain solutions

Examples of these omissions were seen in Category B modules when, for example, a student was trying to find the critical points of a one variable function and omitted a solution in error:

> *'Student was trying to find the critical points of the function $x^4 - 4x^3$ but was unsure how to proceed after differentiating to get 4x^3-3x^2=0. They thought they could cancel the x^2 but this would have led to missing x =0.'*

This was also seen in higher level modules, as the following example of a difficulty experienced by a student in a second-level calculus module:

> *'Student was working on problem finding critical points and couldn't find all of them. Problem was [that] in dividing equation by x+y and not making a new case for x+y=0.'*

Inequalities

The code, *Inequalities,* represented any difficulty where a student showed a lack of understanding of the basic rules for solving inequalities. Particular difficulty was seen with questions on rational inequalities and on applying the triangle inequality. There were just 16 visits for this code. The following is a typical example:

Rational Inequality

A number of students studying a *discrete mathematics* module came to the MSC with a problem where they needed to show where a rational inequality was greater than zero:

> *'Inequalities, student didn't know how -2 < (x-2)/(x+2) was rearranged to 0<(3x+2)/(x+2). Also, why top and bottom line either have to both be negative or both be positive to be greater than zero. Rearranging (3x+2) > 0 to give x >-2/3 and (x + 2) > 0 to give x >-2, why do you pick x >-2/3. Why do you go to the right on the number line when its x > a etc.'*

Sign Rules (+/-)

Eight of the student visits for *Sign Rules* related to a *Hot Topic* organised for a pre-university course. Difficulties experienced by students at higher levels were more likely to be slips in calculations. Students from four Level 1 modules also attended the MSC with *Sign Rule* difficulties. Most errors were basic, for instance, students not knowing or forgetting that two minus signs multiplied together give a plus sign, as seen in the following *tutor entry* relating to a student studying *a* Category A *module*:

> *'Problems with bomdas rules for equations eg 2+3x4-5. Also issues with brackets and plus/minus signs, that / means divides. Went through some examples from class and gave student some more to practice on.'*

Simultaneous Equations

In reporting the *mathematical difficulties* here, we only consider problems with simultaneous equations that did not make use of matrices to find a solution, as we have already covered these in *Matrices.* The following is an example:

> *'Student came in worried about simultaneous equations. I think their main problem was the unfamiliar notation p_1 and p_2 instead of x and y.'*

Grouping 2: Calculus

Critical Points

In this code difficulties where students are asked to find the critical points of functions of one variable were only included. Instances where students are asked to identify critical points given the graph of a function are coded under Graphs. This is an example of a typical entry, where a student was studying the Applied Mathematics module:

> *'How to find the stability of the fixed points of du/dtau=Au/u+1 - u. Tutor also covered getting critical points and used example f(x) = 2x^3 + 15x^2 - 504x + 14 found f'(x)=0 and then showed whether value was max or min point by using value of f''(x). Student had difficulty in finding the critical points of a cubic equation.'*

Domain and Range

In the code *Domain and Range* difficulties arising with understanding the terms and including the concept of well-defined functions were coded. The following is an example of a problem for which four students sought help:

'Finding the domain where the function is undefined eg $\frac{3x}{x-4}$ *is undefined at x = 4'*

Category 7 Other

We have included in the Other Category codes for which visits were low and/or codes that did not fall easily into any one of the other categories.

Table C.1 below gives the list of codes included in this category with the respective number of visits for each.

Other Category	Number
Mathematical expressions	30
Sets	27
Modelling	16
Co-ordinate geometry	14
Pattern spotting	11
Converting units	10

Mathematical expressions

Sets

Elementary understanding of *Sets* was required by students visiting the MSC from five separate modules. Seven of the fourteen *tutor entries* were from one third year module. The student, in these cases, did not appear to be familiar with set properties and notation and the queries were fairly basic in nature.

Co-ordinate Geometry

Tutor entries here were from *Calculus A*, *Calculus B*, *Linear Algebra A* and one third year module. They related to difficulties with linear and circular functions:

> *'Needed help with equations of the circle and parametric equation of the line. Tutor showed r = a + bt and circle centre (4,0) and (point (0,3) on the circumference.'*

Pattern Spotting

This code represents *mathematical difficulties* where students were unable to observe patterns to aid solution to problems. An example of a *mathematical difficulty* in this area is shown where a student in a *discrete mathematics* module had a problem.

> *'Geometric series and finding the sum of a geometric series, difficulty recognizing the pattern, Question was on time opening and closing doors given as 1 min + + 1/2 min + 1/4 min+ ...+ 1/(2^(n-1)) . . .'*

Converting Units

Students from eight modules experienced difficulties for this code. A difficulty in converting from one unit to another is seen in the following example:

> *'Students came in asking how to convert units from for example nano-metres to micro-metres. . . Examples covered: 1.55 km to metres; 0.198g to mg; 1 micro sec to sec; 1 microsec/1 millisec =10^(-6)/ 10^(-3)= 10^(-6 - (-3)) = 10^(-3); 10^(-3)m in a mm; 546 x 10^(-3)m = 5.46 x 10^(-1)m and few other examples of sc. notat'n . . .'*

The *mathematical difficulties* which students experienced in the Category *Other* are either few in number or so varied in mathematical content that it is not possible to categorise them into specific problem areas when addressing our first research question. They are the typical of the once-off problems for which the one-on-one tutoring in the MSC is the ideal solution.

Appendix D

UCD Level Descriptors

Level	Knowledge and Understanding Required
0	Have demonstrated basic knowledge and understanding, underpinned by the basic theories, concepts or methods of the field of study, at a level appropriate to at the transition from secondary to tertiary education and which is typically at a level supported by introductory third-level textbooks.
1	Have demonstrated basic knowledge and understanding, underpinned by the basic theories, concepts or methods of the field of study, that builds upon secondary education and which is typically at a level supported by introductory third-level textbooks.
2	Have demonstrated specialized knowledge and understanding, underpinned by the more advanced theories, concepts or methods of the field of study, have begun to show some awareness of the limitations of current knowledge and the sources of new knowledge, and which is typically supported by intermediate and advanced textbooks.
3	Have demonstrated specialized, detailed or advanced knowledge and understanding, underpinned by advanced theories, concepts or methods, which includes a clear awareness of the limitations of current knowledge and the sources of new knowledge, which is supported by advanced textbooks, but includes some aspects that will be informed by knowledge at the forefront of the field of study.
4	Have demonstrated specialized, detailed or advanced theoretical and conceptual knowledge and understanding, which is based consideration of current debate and controversy at the forefront of the field and that provides a basis or opportunity for originality in developing and/or applying ideas, often within a research context.

Appendix E

Maths Support Centre-Module Coordinator

Partnership Agreement

Rationale: The purpose of this agreement is to facilitate and streamline the communication between the Maths Support Centre (MSC) and Module Coordinators/Lecturers so your students are provided with the best learning experience possible.

1. Do you give permission to MATH1**** students to attend the MSC? Yes No

If yes what level of support do you want the MSC to provide?

Details: e.g. tutors to help with lecture notes but not the assigned homework or Continuous Assessment components

2. Do you agree to me (or an MSC representative) attending one of your lectures to advertise the MSC? Yes No
   When -
   Where -

3. Do you agree to put up a slide/announcement in the first weeks of lectures (and on Blackboard/Moodle) advertising the MSC and how you want your students to use it in terms of assignments, lecture material, notes etc? Yes No

4. Do you agree to set up the MSC manager as a student/tutor on your Blackboard/Moodle page for access to the module's material? Yes No

5. Can you provide the MSC with (approximate) dates of the midterm, CA due?

Details:

6. Do you agree to meet at the end of the semester to review this agreement and discuss your second semester module(s)? Yes No

Agreement: If you agree to the above, the MSC will ask you to engage with the MSC feedback mechanism i.e. you will be sent a weekly email on Friday at 1.30pm with details of the visits to the MSC from the module MATH1 *****.

If a significant proportion of students from this module attend the MSC in a given week the MSC manager will inform the Module Coordinator of the issue.
If these visits continue the manager will inform the Module Coordinator again and if the issue persists we will agree on how to proceed regarding support from the MSC for this module. The MSC manager will ask the Module Coordinator to inform their students of this.

Signature: MSC manager - Anthony Cronin
Signature: MATH1**** Module Coordinator/Lecturer -

The Process

Early September/January - MSC manager meets Module Coordinator (MC) to discuss MSC feedback from previous year and plans for the year ahead.

Same meeting - MSC manager runs through MSC-Module Coordinator Partnership Agreement and discusses any issues arising.

MSC manager and Module Coordinator agree on the partnership document and both sign the MSC-MC agreement.

A copy of the signed agreement will be forwarded to the MC after this meeting.

Notes

1. *The MSC does not open during the examination weeks*
2. *The MSC opens in week one (two) in semester one (two) respectively*
3. *The MSC website is www.ucd.ie/msc*

Appendix F

Schedule for MSC Tutor Focus Group - 15 May, 2015

As experienced tutors who have been working the in the MSC over the last year and recording *tutor entries* on students' visits in the database, we are keen to enlist your help and feedback.

We plan to continue recording *tutor entries* into the future with the aim of feeding back each *topic entry* to the relevant lecturer. Therefore, the first aim of this focus group is to gain insights into your understanding of what a *topic entry* is, how your understanding developed, and whether you feel that the requirement to record these entries has impacted on your practice. The second aim is to gain an insight into how make the process or recording *tutor entries* as efficient and effective as possible. Finally, we would like to have your opinions on the what type of training process we might out in place for new tutors to help them understand what constitutes a *topic entry*, and how to record it as efficiently as possible.

Question 1

What is your understanding of what a *topic entry* is?

- *What is your understanding of the type(s) of information a topic entry should contain?*
- *In your opinion is there a "typical" format for a topic entry?*
- *How would you describe or define the concept of topic entry to a new tutor?*

Question 2

How did your understanding of what constitutes a *topic entry* develop?

- *What did you find most effective in helping you come to an understanding of what a topic entry is?*

Question 3

In your opinion, has the requirement to record a topic entry on each student visit impacted your practice in any way?

i. *Has it impacted on your interaction with the student? If so, in what ways?*
ii. *Has it increased your levels of reflection? If so, in what ways? (On students' difficulties? On your practice?)*

Question 4

In your opinion, what is the most efficient way of inputting tutor entries to the database?

i. *Do you have a preference for typing in entries or or using drop-down menus (assuming all issues with database ironed out)?*
ii. *Do you think it is better to input after each student visit or at the end of a session? Why?*
iii. *On average how long do you think the inputting process takes? Has the new system the potential to shorten this time?*

Question 5

In your opinion, what essential elements should a training process for new tutors contain?

- *What key elements do we need to focus on in educating new tutors?*
- *In your opinion, how long should the training process last?*
- *Do we need a one-to-one element?*
- *In your opinion, would working with tutors to input good tutor entries provide a means of encouraging them to reflect on, and improve, their practice?*

Question 6

We are now at the end of the focus group. Is there anything you'd like to add?